A Supplementary File for "Towards Exploratory Landscape Analysis for Large-scale Optimization: A Dimensionality Reduction Framework"

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ABSTRACT

This paper is a supplementary file for "Towards Exploratory Landscape Analysis for Large-scale Optimization: A Dimensionality Reduction Framework".



Figure S.1: Average computation time (sec) of three sampling methods over 31 runs. For each run, the three sampling methods generate a sample X with the size $50 \times n$ in $[0, 1]^n$, where n is the dimension. IHS is the improved Latin hypercube sampling method (lhs in the flacco package). LHS is the Latin hypercube sampling method (lhs in the pyDOE package). RND is the random number generator in the Python Numpy library. We stopped the run of IHS for n = 320 because the single-run computation of IHS did not finish within one day.



Figure S.2: Average feature computation time (sec) on the first instance of f_1 with $n \in \{2, 3, 5, 10, 20, 40, 80, 160, 320, 640\}$. The sample size |X| was set to 100 for all dimensions. In contrast to Figure 1 in the main paper, this figure shows how the feature computation time scale with respect to the dimension n.



Figure S.3: Average feature computation time (sec) on the first instance of f_1 with n = 2. The sample size |X| was set to 100, 150, 250, 500, 1000, 2000, 8000, 16000, 32000, where they correspond to $50 \times 2, 50 \times 3, 50 \times 5, 50 \times 10, 50 \times 20, 50 \times 40, 50 \times 80, 50 \times 160, 50 \times 320, 50 \times 640$, respectively. In contrast to Figure 1 in the main paper, this figure shows how the feature computation time scale with respect to the sample size |X|.



Figure S.4: Average feature computation time (sec) on the first instance of f_1 with $n \in \{2, 3, 5, 10, 20, 40, 80, 160, 320, 640\}$. This figure is an extended version of Figure 1 in the main paper by including the results of the dimensionality reduction versions of the four cell mapping features (d_gcm, d_cm_angle, d_cm_conv, d_cm_grad).

Table S.1: Average accuracy of C7, C7-E2, and C7-D2 on the 24 BBOB functions with $n \in \{2, 3, 5, 10, 20, 40, 80, 160, 320, 640\}$. The numbers in parentheses show the standard deviation.

	(a) M	fultimodality		(b) Global structure					
	C7	C7-E2	C7-D2		C7	C7-E2	C7-D2		
2	0.642 (0.404)	0.703 (0.357)	Na	2	0.758 (0.402)	0.786 (0.374)	Na		
3	0.569(0.432)	0.597(0.419)	0.578 (0.437)	3	0.792 (0.381)	0.797(0.373)	0.778 (0.389)		
5	0.536(0.443)	0.625 (0.411)	0.622 (0.415)	5	0.769 (0.395)	0.772(0.407)	0.761 (0.412)		
10	0.522 (0.451)	0.631 (0.405)	0.647 (0.434)	10	0.708 (0.424)	0.750 (0.425)	0.692 (0.435)		
20	0.531 (0.427)	0.725 (0.397)	0.639 (0.413)	20	0.714 (0.426)	0.742 (0.419)	0.697 (0.440)		
40	0.556 (0.465)	0.689 (0.394)	0.617 (0.441)	40	0.711 (0.460)	0.711 (0.443)	0.711 (0.460)		
80	0.600 (0.450)	0.783 (0.371)	0.622 (0.454)	80	0.697 (0.444)	0.697 (0.449)	0.697 (0.453)		
160	0.522 (0.457)	0.650 (0.438)	0.561 (0.475)	160	0.647 (0.447)	0.650 (0.457)	0.631 (0.463)		
320	0.544 (0.472)	Na	0.578 (0.472)	320	0.664 (0.474)	Na	0.667 (0.482)		
640	0.514 (0.485)	Na	0.583 (0.489)	640	0.667 (0.461)	Na	0.678(0.468)		

	(c)	Separability			(d) Variable scaling						
	C7	C7-E2	C7-D2		C7	C7-E2	C7-D2				
2	0.756 (0.381)	0.803 (0.344)	Na	2	0.581 (0.496)	0.586 (0.495)	Na				
3	0.789 (0.363)	0.856 (0.336)	0.786 (0.387)	3	0.611 (0.445)	0.583 (0.468)	0.611 (0.456)				
5	0.758 (0.395)	0.864 (0.322)	0.761 (0.421)	5	0.628 (0.454)	0.600 (0.465)	0.594 (0.480)				
10	0.758 (0.401)	0.828 (0.344)	0.753 (0.411)	10	0.508(0.489)	0.556 (0.494)	0.572 (0.496)				
20	0.708(0.427)	0.808(0.377)	0.744 (0.401)	20	0.539 (0.507)	0.578 (0.499)	0.583(0.504)				
40	0.767 (0.389)	0.836 (0.349)	0.758 (0.401)	40	0.539 (0.507)	0.581 (0.496)	0.556 (0.492)				
80	0.703 (0.440)	0.775 (0.405)	0.686 (0.447)	80	0.542 (0.509)	0.583 (0.504)	0.581 (0.501)				
160	0.697 (0.456)	0.753 (0.421)	0.667 (0.476)	160	0.558 (0.497)	0.583 (0.504)	0.583(0.504)				
320	0.722 (0.441)	Na	0.669 (0.467)	320	0.547 (0.499)	Na	0.583(0.504)				
640	0.725 (0.400)	Na	0.669 (0.453)	640	0.542 (0.509)	Na	0.542 (0.509)				

(e) Homogeneity

(f) Basin size

	C7	C7-E2	C7-D2		C7	C7-E2	C7-D2
2	0.633 (0.397)	0.636 (0.395)	Na	2	0.450 (0.421)	0.492 (0.409)	Na
3	0.664 (0.378)	0.608 (0.379)	0.608 (0.389)	3	0.544 (0.430)	0.547 (0.400)	0.522 (0.428)
5	0.647 (0.399)	0.578 (0.431)	0.647 (0.408)	5	0.508(0.404)	0.628(0.343)	0.522 (0.373)
10	0.722 (0.380)	0.658 (0.399)	0.769 (0.338)	10	0.494 (0.419)	0.608 (0.387)	0.636 (0.384)
20	0.772 (0.369)	0.731 (0.400)	0.731 (0.403)	20	0.467 (0.429)	0.647 (0.398)	0.558 (0.409)
40	0.761(0.385)	0.700 (0.432)	0.744 (0.407)	40	0.531 (0.447)	0.617(0.412)	0.614 (0.423)
80	0.703 (0.450)	0.639 (0.448)	0.742 (0.412)	80	0.464(0.474)	0.650(0.415)	0.597(0.460)
160	0.656(0.477)	0.625 (0.466)	0.683 (0.439)	160	0.472 (0.457)	0.578(0.460)	0.614 (0.447)
320	0.719 (0.438)	Na	0.761 (0.417)	320	0.408(0.474)	Na	0.508(0.446)
640	0.772 (0.413)	Na	0.781 (0.411)	640	0.456 (0.477)	Na	0.528 (0.471)

	(g)	GL contrast		(h) Overall average						
	C7	C7-E2	C7-D2		C7	C7-E2	C7-D2			
2	0.572 (0.411)	0.642 (0.335)	Na	2	0.627 (0.422)	0.664 (0.396)	Na			
3	0.586 (0.429)	0.600 (0.393)	0.572 (0.422)	3	0.651 (0.413)	0.656(0.406)	0.637 (0.420)			
5	0.497 (0.411)	0.669 (0.358)	0.536 (0.411)	5	0.621 (0.421)	0.677 (0.399)	0.635 (0.420)			
10	0.542 (0.403)	0.656(0.368)	0.619 (0.410)	10	0.608(0.431)	0.669 (0.407)	0.670(0.415)			
20	0.544(0.424)	0.728(0.377)	0.617 (0.384)	20	0.611 (0.437)	0.708(0.410)	0.653 (0.421)			
40	0.533 (0.422)	0.708(0.368)	0.597 (0.428)	40	0.628 (0.445)	0.692 (0.415)	0.657 (0.436)			
80	0.558(0.458)	0.703 (0.423)	0.619 (0.458)	80	0.610(0.461)	0.690 (0.430)	0.649 (0.451)			
160	0.553 (0.441)	0.636 (0.453)	0.622 (0.445)	160	0.587(0.460)	0.639 (0.453)	0.623 (0.458)			
320	0.550 (0.478)	Na	0.647 (0.436)	320	0.594 (0.472)	Na	0.631 (0.459)			
640	0.572 (0.481)	Na	0.614(0.473)	640	0.607(0.467)	Na	0.628(0.467)			

Table S.2: Average accuracy of C7, C7-C4, and C7-D4 on the 24 BBOB functions with $n \in \{2, 3, 5, 10, 20, 40, 80, 160, 320, 640\}$. The numbers in parentheses show the standard deviation.

	(a) M	fultimodality		(b) Global structure					
	C7	C7-C4	C7-D4		C7	C7-C4	C7-D4		
2	0.642 (0.404)	0.725 (0.368)	Na	2	0.758 (0.402)	0.789 (0.359)	Na		
3	0.569 (0.432)	0.622 (0.413)	0.547 (0.424)	3	0.792 (0.381)	0.797 (0.368)	0.792 (0.392)		
5	0.536 (0.443)	0.633 (0.417)	0.522 (0.423)	5	0.769 (0.395)	0.783 (0.396)	0.778(0.400)		
10	0.522 (0.451)	Na	0.561 (0.414)	10	0.708(0.424)	Na	0.719 (0.426)		
20	0.531 (0.427)	Na	0.550 (0.436)	20	0.714 (0.426)	Na	0.733 (0.416)		
40	0.556 (0.465)	Na	0.564 (0.449)	40	0.711 (0.460)	Na	0.714 (0.456)		
80	0.600 (0.450)	Na	0.544 (0.436)	80	0.697 (0.444)	Na	0.706 (0.440)		
160	0.522 (0.457)	Na	0.528 (0.445)	160	0.647 (0.447)	Na	0.656 (0.453)		
320	0.544 (0.472)	Na	0.542 (0.472)	320	0.664 (0.474)	Na	0.664 (0.474)		
640	0.514 (0.485)	Na	0.536 (0.466)	640	0.667 (0.461)	Na	0.683 (0.461)		

	(c)	Separability			(d) Variable scaling					
	C7	C7-C4	C7-D4		C7	C7-C4	C7-D4			
2	0.756 (0.381)	0.811 (0.345)	Na	2	0.581 (0.496)	0.586 (0.490)	Na			
3	0.789 (0.363)	0.856 (0.337)	0.800 (0.390)	3	0.611 (0.445)	0.592 (0.472)	0.650(0.457)			
5	0.758 (0.395)	0.825 (0.352)	0.761 (0.409)	5	0.628 (0.454)	0.597 (0.462)	0.636 (0.469)			
10	0.758 (0.401)	Na	0.758 (0.414)	10	0.508(0.489)	Na	0.528 (0.486)			
20	0.708(0.427)	Na	0.731 (0.410)	20	0.539 (0.507)	Na	0.536 (0.504)			
40	0.767 (0.389)	Na	0.775 (0.402)	40	0.539 (0.507)	Na	0.533(0.490)			
80	0.703(0.440)	Na	0.719 (0.425)	80	0.542 (0.509)	Na	0.581 (0.501)			
160	0.697 (0.456)	Na	0.708(0.440)	160	0.558 (0.497)	Na	0.583(0.504)			
320	0.722 (0.441)	Na	0.731 (0.433)	320	0.547 (0.499)	Na	0.544 (0.506)			
640	0.725 (0.400)	Na	0.744 (0.413)	640	0.542 (0.509)	Na	0.542 (0.509)			

(e) Homogeneity

(f) Basin size

	C7	C7-C4	C7-D4		C7	C7-C4	C7-D4
2	0.633 (0.397)	0.650 (0.392)	Na	2	0.450 (0.421)	0.492 (0.403)	Na
3	0.664(0.378)	0.617 (0.364)	0.617 (0.397)	3	0.544 (0.430)	0.539 (0.389)	0.550 (0.434)
5	0.647 (0.399)	0.564 (0.435)	0.636 (0.412)	5	0.508(0.404)	0.658 (0.324)	0.478 (0.403)
10	0.722 (0.380)	Na	0.706 (0.395)	10	0.494 (0.419)	Na	0.531 (0.388)
20	0.772 (0.369)	Na	0.769 (0.367)	20	0.467 (0.429)	Na	0.483 (0.425)
40	0.761 (0.385)	Na	0.781 (0.368)	40	0.531 (0.447)	Na	0.533 (0.440)
80	0.703(0.450)	Na	0.706 (0.434)	80	0.464(0.474)	Na	0.519 (0.448)
160	0.656(0.477)	Na	0.656(0.465)	160	0.472 (0.457)	Na	0.472 (0.459)
320	0.719 (0.438)	Na	0.772 (0.408)	320	0.408(0.474)	Na	0.400 (0.478)
640	0.772(0.413)	Na	0.758 (0.424)	640	0.456 (0.477)	Na	0.492 (0.475)

(g) GL contrast

(h) Overall average

	C7	C7-C4	C7-D4		C7	C7-C4	C7-D4
2	0.572 (0.411)	0.647 (0.362)	Na	2	0.627 (0.422)	0.671 (0.398)	Na
3	0.586 (0.429)	0.608 (0.392)	0.578 (0.423)	3	0.651 (0.413)	0.662(0.401)	0.648(0.422)
5	0.497 (0.411)	0.689 (0.363)	0.503 (0.407)	5	0.621 (0.421)	0.679 (0.398)	0.616 (0.426)
10	0.542 (0.403)	Na	0.592 (0.419)	10	0.608 (0.431)	Na	0.628(0.423)
20	0.544(0.424)	Na	0.558 (0.410)	20	0.611 (0.437)	Na	0.623 (0.432)
40	0.533 (0.422)	Na	0.522 (0.417)	40	0.628 (0.445)	Na	0.632(0.440)
80	0.558(0.458)	Na	0.553 (0.446)	80	0.610(0.461)	Na	0.618(0.447)
160	0.553 (0.441)	Na	0.547 (0.440)	160	0.587(0.460)	Na	0.593(0.457)
320	0.550 (0.478)	Na	0.550 (0.473)	320	0.594(0.472)	Na	0.600(0.471)
640	0.572 (0.481)	Na	0.544 (0.486)	640	0.607(0.467)	Na	0.614(0.466)

able S.3: Average rankings of the d_(ela_meta (dem) and d_ela_level (del) features in C7-D2 for $n \in \{$	3, 5, 10, 20, 40, 80, 160, 320, 640 }.
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(a) <i>n</i> = 3		(b) <i>n</i> = 5		(c) $n = 10$		(d) $n = 20$	
Feature	Rank	Feature	Rank	Feature	Rank	Feature	Rank
dem.lin_simple.coef.max	10.9	dem.lin_simple.coef.max	8.9	dem.lin_simple.coef.max	9.1	dem.lin_simple.coef.max	8.0
dem.lin_simple.intercept	14.2	dem.lin_simple.intercept	9.6	dem.lin_simple.intercept	11.5	dem.lin_simple.intercept	11.1
dem.lin_simple.coef.min	17.0	dem.quad_simple.adj_r2	17.9	dem.quad_w_interact.adj_r2	16.8	dem.quad_w_interact.adj_r2	13.8
dem.quad_simple.adj_r2	18.3	dem.lin_simple.coef.min	18.8	dem.lin_simple.adj_r2	17.0	dem.lin_w_interact.adj_r2	16.3
dem.quad_w_interact.adj_r2	18.6	dem.quad_w_interact.adj_r2	20.4	dem.lin_w_interact.adj_r2	17.1	dem.lin_simple.adj_r2	16.7
dem.lin_simple.adj_r2	25.2	dem.lin_w_interact.adj_r2	20.4	dem.quad_simple.adj_r2	17.4	dem.quad_simple.adj_r2	16.7
dem.lin_w_interact.adj_r2	27.2	dem.lin_simple.adj_r2	21.0	dem.lin_simple.coef.min	19.8	dem.lin_simple.coef.min	16.9
del.costs_runtime	36.7	del.mmce_lda_25	40.8	del.mmce_lda_25	43.2	del.mmce_mda_50	33.4
del.mmce_mda_25	43.6	del.mmce_lda_10	42.5	del.mmce_lda_10	45.0	del.mmce_lda_25	34.4
dem.quad_simple.cond	44.0	del.mmce_mda_50	46.1	del.mmce_mda_25	45.4	del.mmce_mda_25	35.6
del.mmce_mda_10	48.0	del.mmce_mda_10	46.8	del.mmce_mda_10	45.7	del.mmce_lda_10	38.2
del.mmce_lda_25	48.4	del.mmce_lda_50	47.4	del.costs_runtime	46.6	del.mmce_mda_10	38.2
del.qda_mda_25	49.5	del.costs_runtime	48.8	del.mmce_mda_50	46.9	del.mmce_lda_50	44.4
del.mmce_qda_25	50.5	del.mmce_mda_25	50.4	del.mmce_lda_50	49.0	del.qda_mda_10	50.9
dem.lin_simple.coef.max_by_min	51.9	del.lda_qda_10	51.5	dem.costs_runtime	50.4	del.lda_qda_25	51.0
del.mmce_lda_50	51.9	del.lda_mda_50	52.6	del.lda_qda_50	52.2	del.lda_mda_50	51.1
del.qda_mda_10	52.3	dem.quad_simple.cond	55.2	del.qda_mda_10	52.4	del.lda_qda_10	51.3
del.lda_qda_50	53.8	del.mmce_qda_25	57.6	del.lda_mda_50	52.8	del.mmce_qda_25	52.0
del.mmce_lda_10	54.7	del.qda_mda_25	57.8	del.lda_qda_10	56.3	del.lda_qda_50	53.3
del.lda_qda_10	55.4	del.lda_qda_50	58.1	del.qda_mda_50	56.3	del.qda_mda_50	54.3
del.lda_mda_50	56.3	del.lda_qda_25	58.7	del.mmce_qda_50	61.4	del.qda_mda_25	56.2
del.lda_qda_25	58.9	dem.costs_runtime	60.3	del.lda_mda_25	61.6	del.mmce_qda_50	59.6
del.qda_mda_50	59.2	del.qda_mda_10	60.4	del.mmce_qda_25	63.1	del.costs_runtime	60.2
del.mmce_mda_50	59.9	del.qda_mda_50	61.6	dem.quad_simple.cond	63.6	dem.lin_simple.coef.max_by_min	61.3
del.lda_mda_10	64.7	dem.lin_simple.coef.max_by_min	63.1	del.lda_qda_25	63.8	del.lda_mda_25	61.5
del.lda_mda_25	64.8	del.mmce_qda_10	63.7	dem.lin_simple.coef.max_by_min	64.2	dem.quad_simple.cond	64.1
del.mmce_qda_50	67.5	del.lda_mda_25	65.8	del.mmce_qda_10	66.1	del.mmce_qda_10	65.2
del.mmce_qda_10	68.4	del.mmce_qda_50	67.8	del.qda_mda_25	67.3	dem.costs_runtime	72.0
dem.costs_runtime	70.2	del.lda_mda_10	70.5	del.lda_mda_10	72.7	del.lda_mda_10	74.6

(e) $n = 40$		(f) $n = 80$		(g) $n = 160$		(h) $n = 320$	
Feature	Rank	Feature	Rank	Feature	Rank	Feature	Rank
dem.lin_simple.coef.max	9.3	dem.lin_simple.coef.max	8.1	dem.lin_simple.coef.max	7.6	dem.lin_simple.coef.max	6.2
dem.lin_simple.intercept	10.3	dem.lin_simple.intercept	10.1	dem.lin_simple.intercept	11.3	dem.lin_simple.intercept	13.6
dem.quad_w_interact.adj_r2	14.1	dem.quad_w_interact.adj_r2	17.4	dem.quad_w_interact.adj_r2	18.7	dem.lin_w_interact.adj_r2	21.0
<pre>dem.lin_w_interact.adj_r2</pre>	16.9	dem.quad_simple.adj_r2	18.5	dem.lin_w_interact.adj_r2	19.5	dem.lin_simple.coef.min	21.4
dem.lin_simple.adj_r2	17.4	dem.lin_simple.adj_r2	18.7	dem.lin_simple.adj_r2	20.5	dem.quad_w_interact.adj_r2	23.1
<pre>dem.quad_simple.adj_r2</pre>	19.8	<pre>dem.lin_w_interact.adj_r2</pre>	19.0	dem.quad_simple.adj_r2	21.1	dem.lin_simple.adj_r2	23.6
dem.lin_simple.coef.min	20.4	dem.lin_simple.coef.min	24.2	dem.lin_simple.coef.min	22.6	<pre>dem.quad_simple.adj_r2</pre>	24.4
del.mmce_mda_50	35.6	del.mmce_mda_50	34.8	del.mmce_mda_50	28.6	del.mmce_lda_25	36.6
del.mmce_mda_10	39.4	del.mmce_lda_25	36.6	del.mmce_lda_25	35.6	del.mmce_lda_10	36.7
del.mmce_lda_10	39.6	del.mmce_lda_10	38.9	del.mmce_lda_50	38.4	del.mmce_mda_50	37.0
del.mmce_lda_50	40.8	del.mmce_mda_10	39.9	del.mmce_mda_25	39.4	del.mmce_qda_25	37.2
del.mmce_mda_25	42.7	del.mmce_mda_25	42.1	del.mmce_lda_10	40.9	del.mmce_mda_10	38.1
del.mmce_lda_25	43.2	del.mmce_lda_50	44.1	del.mmce_mda_10	41.4	del.mmce_lda_50	38.6
del.lda_qda_50	45.7	del.qda_mda_50	44.9	del.mmce_qda_25	47.2	del.mmce_mda_25	42.0
del.qda_mda_50	46.9	del.lda_qda_50	45.0	del.mmce_qda_50	49.7	del.qda_mda_50	46.9
del.lda_mda_50	48.7	del.mmce_qda_10	50.1	del.qda_mda_50	49.7	del.costs_runtime	48.8
del.lda_qda_25	50.9	del.lda_mda_50	50.8	del.mmce_qda_10	51.8	del.mmce_qda_10	49.4
del.qda_mda_10	54.8	del.mmce_qda_25	52.5	del.lda_mda_50	52.8	del.lda_qda_50	51.0
del.mmce_qda_10	56.3	del.costs_runtime	54.9	del.lda_qda_25	53.2	del.mmce_qda_50	54.9
del.lda_qda_10	56.6	del.lda_qda_25	55.8	del.lda_qda_50	54.8	del.lda_qda_25	55.0
del.mmce_qda_25	56.8	del.qda_mda_25	57.3	del.qda_mda_10	55.6	del.lda_mda_50	55.4
del.qda_mda_25	57.8	del.lda_qda_10	58.8	del.qda_mda_25	56.1	del.lda_qda_10	56.7
del.lda_mda_25	60.7	del.qda_mda_10	60.0	del.lda_qda_10	56.3	del.qda_mda_25	57.5
del.mmce_qda_50	60.8	del.mmce_qda_50	61.1	del.lda_mda_25	61.3	del.qda_mda_10	58.0
del.costs_runtime	66.2	del.lda_mda_25	62.1	del.costs_runtime	66.1	del.lda_mda_25	59.7
<pre>dem.lin_simple.coef.max_by_min</pre>	67.3	dem.lin_simple.coef.max_by_min	70.1	dem.quad_simple.cond	67.2	dem.costs_runtime	68.7
dem.quad_simple.cond	67.3	dem.quad_simple.cond	72.3	dem.lin_simple.coef.max_by_min	67.3	<pre>dem.lin_simple.coef.max_by_min</pre>	1 69.9
dem.costs_runtime	72.3	dem.costs_runtime	73.9	dem.costs_runtime	71.8	dem.quad_simple.cond	70.1
del.lda_mda_10	75.8	del.lda_mda_10	75.7	del.lda_mda_10	75.8	del.lda_mda_10	74.7

(i) n = 640 (part 1)

(j) n = 640 (part 2)

Feature	Rank	Feature	Rank
dem.lin_simple.coef.max	10.4	del.gda_mda_50	46.2
dem.lin_simple.intercept	14.0	del.mmce_qda_50	46.8
dem.lin_simple.coef.min	22.7	del.lda_qda_50	51.1
dem.quad_simple.adj_r2	22.8	del.lda_qda_25	51.3
dem.quad_w_interact.adj_r2	23.2	del.qda_mda_10	53.1
dem.lin_simple.adj_r2	23.9	del.lda_mda_50	53.8
<pre>dem.lin_w_interact.adj_r2</pre>	25.5	del.lda_qda_10	55.2
del.mmce_qda_25	29.7	del.qda_mda_25	56.3
del.mmce_lda_50	29.7	del.lda_mda_25	64.1
del.mmce_mda_50	30.1	dem.costs_runtime	68.9
del.mmce_lda_25	34.7	dem.lin_simple.coef.max_by_min	69.5
del.mmce_lda_10	36.7	dem.quad_simple.cond	69.6
del.mmce_mda_10	37.3	del.costs_runtime	71.7
del.mmce_qda_10	43.3	del.lda_mda_10	75.1
del.mmce_mda_25	43.4		

Table S.4: Average rankings of the d_cm_angle (dca), d_cm_conv (dcc), d_cm_grad (dcg), and d_gcm (dg) features in C7-D4 for $n \in \{3, 5, 10, 20, 40, 80, 160, 320, 640\}$.

(a) $n = 3$		(b) <i>n</i> = 5	5 (c) <i>n</i> = 10			(d) $n = 20$	
Feature	Rank	Feature	Rank	Feature	Rank	Feature	Rank
dca.costs_runtime	25.4	dca.y_ratio_best2worst.mean	20.4	dca.y_ratio_best2worst.mean	25.4	dca.y_ratio_best2worst.mean	26.1
dca.y_ratio_best2worst.mean	29.6	dca.y_ratio_best2worst.sd	30.8	dca.y_ratio_best2worst.sd	28.9	dca.y_ratio_best2worst.sd	31.6
dca.y_ratio_best2worst.sd	39.0	dca.angle.sd	44.2	dcg.mean	46.3	dca.costs_runtime	46.3
dg.min.costs_runtime	39.0	dca.angle.mean	47.6	dg.near.basin_prob.min	47.1	dca.dlst_ctr2worst.mean	47.3
dg.near.costs runtime	43.9	dca.dist_ctr2worst.sd	48.5	dca.angle.mean	51.2	dca.dist_ctr2worst.sd	51.0
dca.dist_ctr2best.mean	44.5	dcg.sd	49.8	dca.dist_ctr2worst.mean	51.8	dcg.sd	52.0
dca.dist_ctr2worst.sd	48.5	dca.dist_ctr2best.sd	50.4	dg.mean.basin_prob.min	52.5	dcg.costs_runtime	52.8
dcg.mean	48.8	dca.dist_ctr2best.mean	51.9	dcg.sd	53.1	dca.dist_ctr2best.sd	53.4
dg.mean.costs_runtime	48.8	dcg.mean	53.0 53.0	dca.dist_ctr2best.sd	53.2 53.8	dca.angle.mean	53.4 54.6
dca.angle.mean	50.4	dg.near.basin_prob.min	54.8	dca.dist_ctr2best.mean	55.8 54.1	dcg.mean	55.2
dca.dist_ctr2worst.mean	50.8	dcg.costs_runtime	56.9	dg.near.best_attr.prob	56.0	dg.near.basin_prob.min	55.8
dca.angle.sd	52.4	dg.near.best_attr.prob	59.4	dcg.costs_runtime	60.3	dg.mean.basin_prob.min	56.4
dcg.sd	52.8	dg.min.basin_prob.min	60.3	dg.min.basin_prob.min	62.2	dg.near.costs_runtime	58.2
dg.mean.basin_prob.min	61.5	dcc.costs_runtime	61.1	dg.mean.best_attr.prob	65.8	dg.mean.costs_runtime	59.4
dg near basin prob min	63.1	dcc_convex_soft	61.9	dg mean basin prob median	66.8	dg mean basin prob max	65.0
dg.mean.best_attr.prob	63.9	dg.near.basin prob.max	64.5	dg.near.costs_runtime	66.9	dg.mean.best_attr.prob	66.1
dg.min.best_attr.prob	64.6	dg.min.basin_prob.max	65.0	dg.min.basin_prob.max	67.9	dg.min.costs_runtime	67.2
dg.near.basin_prob.max	65.2	dg.min.best_attr.prob	67.6	dg.min.best_attr.prob	67.9	dg.min.basin_prob.min	69.0
dg.min.basin_prob.min	65.7	dg.near.costs_runtime	68.6	dcc.concave.hard	68.7	dg.min.basin_prob.max	69.6
dg.mean.basin_prob.max	66.2	dca.costs_runtime	69.0 70.0	dg.mean.costs_runtime	69.7 70.1	dg.min.best_attr.prob	69.7 70.2
dcc.convex soft	68.8	dg mean best attr prob	70.0	dcc concave soft	70.1	dg near basin prob max	70.2
dg.min.basin prob.max	69.4	dg.mean.basin prob.median	72.9	dg.near.basin prob.max	70.1	dg.mean.basin_prob.max	72.0
dg.mean.basin_prob.median	70.1	dg.near.basin_uncertain.max	75.0	dcc.costs_runtime	72.0	dg.near.basin_certain.sum	72.6
dcc.costs_runtime	71.3	dcc.concave.hard	75.3	dg.near.basin_uncertain.min	72.5	dg.near.uncertain	73.5
dcc.concave.hard	77.0	dg.min.basin_prob.median	76.7	dg.min.costs_runtime	73.8	dg.near.best_attr.prob	75.1
dg.near.basin_prob.median	77.6	dg.mean.costs_runtime	77.5	dca.costs_runtime	73.9	dg.mean.basin_uncertain.min	82.0
dg mean basin uncertain mean	78.2 81.0	dg near basin prob median	80.7 81.1	dg mean basin uncertain mean	70.5 77 4	da mean basin uncertain median	82.0 82.0
dg.mean.basin_prob.mean	81.2	dg.min.basin_prob.median	81.5	dg.mean.basin_prob.mean	78.1	dg.mean.basin_certain.sum	82.6
dg.mean.basin_uncertain.median	83.8	dg.min.basin_uncertain.mean	81.5	dg.mean.basin_certain.sum	81.2	dcc.convex.soft	82.6
dg.near.basin_certain.mean	84.2	dg.mean.basin_uncertain.mean	81.6	dg.near.basin_certain.sum	83.5	dg.mean.basin_uncertain.mean	83.0
dg.min.basin_uncertain.median	85.3	dg.mean.basin_prob.mean	83.1	dg.near.uncertain	84.2	dg.near.basin_certain.median	84.1
dg.min.basin_uncertain.mean	85.8	dg.near.uncertain	83.4	dg.mean.basin_uncertain.min	85.4	dg.mean.basin_prob.mean	84.8
dg.min.basin_prob.mean	86.9	dg.near.basin_certain.mean	83.7	dg.near.basin_uncertain.max	86.9	dcc.concave.hard	85.9
dg min basin uncertain min	88 7	dg mean basin certain sum	85 0	dg near basin prob median	87.8	dg near basin prob median	80.7 87.4
dcc.convex.hard	88.8	dg.mean.basin_uncertain.median	86.4	dg.near.basin_certain.mean	88.0	dg.min.basin_uncertain.max	88.4
dg.min.basin_certain.sum	89.0	dg.min.basin_uncertain.median	86.5	dg.mean.uncertain	89.9	dg.near.basin_certain.min	89.3
dg.near.uncertain	89.1	dg.mean.basin_uncertain.min	87.1	dg.mean.basin_certain.mean	91.2	dg.min.basin_prob.median	89.5
dg.near.basin_certain.sum	89.3	dg.near.basin_uncertain.min	87.8	dg.min.basin_uncertain.max	92.7	dg.mean.uncertain	90.2
dg.min.basin_certain.mean	89.9	dg.near.basin_certain.max	87.8	dg.mean.basin_uncertain.max	93.3	dg.near.basin_uncertain.max	92.7
dg near basin uncertain max	91.9	dg min basin certain mean	95.0 95.7	dg min basin uncertain mean	94.0	dg mean basin uncertain max	93.0 93.4
dg.mean.basin_uncertain.max	93.1	dcc.convex.hard	96.1	dg.min.basin_prob.mean	94.0	dg.mean.basin_certain.mean	95.2
dg.mean.basin_certain.mean	93.2	dg.mean.basin_uncertain.max	96.9	dg.near.basin_certain.max	94.4	dg.min.basin_uncertain.mean	95.6
dg.min.basin_uncertain.max	93.3	dg.mean.basin_certain.mean	97.0	dg.mean.basin_certain.median	95.0	dg.min.basin_prob.mean	96.2
dg.min.basin_certain.max	96.6	dg.near.basin_certain.median	97.1	dg.mean.basin_certain.max	96.4	dg.min.basin_certain.mean	96.9
dg.min.basin_certain.median	96.9	dg.min.basin_uncertain.min	97.5	dg.min.basin_certain.sum	98.7	dg.near.basin_certain.max	97.2
dg near basin uncertain min	98.1	dg min basin certain sum	98.5	dg min uncertain	99.2	dg min basin certain sum	99.5
dg.mean.basin_certain.max	98.2	dg.mean.basin_certain.max	99.0	dcc.convex.hard	99.7	dg.mean.basin_certain.median	99.7
dg.mean.uncertain	98.5	dg.min.basin_certain.max	100.0	dg.near.basin_certain.median	101.2	dg.min.uncertain	99.8
dg.near.basin_uncertain.median	98.8	dg.min.basin_certain.median	102.0	dg.min.basin_certain.mean	101.3	dcc.convex.hard	100.2
dg.mean.tcells	99.5	dg.min.uncertain	102.0	dg.near.basin_uncertain.median	101.8	dg.min.basin_certain.max	100.4
dg.mean.basin_uncertain.min	100.0	dg.mean.basin_certain.median	102.5	dg.min.basin_certain.median	103.9	dg.min.basin_uncertain.min	101.3
dg mean basin certain median	100.5	dg mean tcells	105.1	dg near basin uncertain mean	104.0	dg near basin uncertain median	101.4
dg.min.tcells	104.0	dg.near.basin_uncertain.mean	105.6	dg.min.basin_certain.max	104.5	dg.mean.tcells	105.4
dg.near.basin_uncertain.mean	107.8	dg.near.attractors	106.0	dg.mean.tcells	104.7	dg.mean.pcells	111.4
dg.near.basin_prob.mean	109.1	dg.min.tcells	106.1	dg.near.attractors	105.1	dg.min.tcells	111.4
dg.near.attractors	109.5	dg.near.basin_prob.mean	106.7	dg.near.tcells	105.4	dg.near.basin_prob.mean	111.8
dg.mean.pcells	110.5	dg.near.tcells	108.1	dg.min.tcells	108.7	dg.near.basin_uncertain.mean	112.2
dg near tcells	110.0	dg mean ncells	108.5	dg near basin certain min	100.7	dg near attractors	112.4
dg.min.basin_certain.min	111.1	dg.mean.attractors	111.1	dg.mean.attractors	112.6	dg.mean.basin_certain.min	113.1
dg.mean.basin_uncertain.sum	111.1	dg.mean.basin_uncertain.sum	112.0	dg.mean.pcells	112.7	dg.near.tcells	113.3
dg.mean.attractors	111.3	dg.min.basin_uncertain.sum	113.2	dg.mean.basin_certain.min	112.8	dg.mean.basin_uncertain.sum	113.3
dg.min.basin_uncertain.sum	112.6	dg.mean.basin_certain.min	114.2	dg.min.basin_certain.min	113.4	dg.near.pcells	113.5
dg.near.basin_certain.min	115.2	dg.near.basin_certain.min	114.4	dg.mean.basin_uncertain.sum	113.8	dg.min.basin_uncertain.sum	113.6
ug.mean.basin_certain.min	11/.8	dg min pcells	116.1	dg min attractors	114.5	ug.min.pasin_certain.min	116.1
dg.min.attractors	120.3	dg.min.attractors	118.6	dg.min.pcells	120.5	dg.min.attractors	120.9
dg.near.best_attr.no	123.5	dg.near.best_attr.no	123.7	dg.near.best_attr.no	124.0	dg.min.pcells	121.3

Table S.5: Average rankings of the d_cm_angle (dca), d_cm_conv (dcc), d_cm_grad (dcg), and d_gcm (dg) features in C7-D4 for $n \in \{40, 80, 160, 320, 640\}$.

(a) $n = 40$		(b) <i>n</i> = 80		(c) $n = 160$		(d) <i>n</i> = 320	
Feature	Rank	Feature	Rank	Feature	Rank	Feature	Rank
dca.y_ratio_best2worst.mean	34.2	dca.y_ratio_best2worst.mean	40.5	dca.y_ratio_best2worst.mean	43.4	dca.y_ratio_best2worst.sd	43.2
dca.y_ratio_best2worst.sd	36.0	dca.y_ratio_best2worst.sd	44.6	dca.y_ratio_best2worst.sd	45.2	dca.y_ratio_best2worst.mean	46.5
dg.mean.basin_prob.min	49.0	dca.angle.mean	48.9	dcc.costs_runtime	47.4	dca.angle.mean	49.2
dcg sd	50.5 53.1	dg mean basin prob min	54.4 54.9	dcg sd	40.4 50.6	dg near costs runtime	49.5
dcg.mean	53.7	dca.dist_ctr2best.mean	55.3	dca.angle.sd	51.6	dca.angle.sd	51.2
dg.mean.basin_prob.median	54.5	dcg.sd	55.4	dcg.mean	51.6	dcg.costs_runtime	51.8
dca.angle.mean	54.8	dcg.costs_runtime	56.7	dcg.costs_runtime	55.2	dca.dist_ctr2worst.mean	52.7
dca.dlst_ctr2worst.mean	55.5	dcg.mean	58.4	dg.mean.basin_prob.min	56.6	dca.dist_ctr2best.sd	53.6
dcg costs runtime	57.8	dca dist ctr2worst sd	59.9	dca dist ctr2worst sd	58.0 58.7	dcg mean	55.9
dg.near.basin_prob.min	59.2	dca.dist_ctr2worst.mean	60.0	dg.near.basin_prob.max	60.3	dca.dist_ctr2best.mean	56.3
dcc.costs_runtime	59.7	dca.dist_ctr2best.sd	60.2	dg.min.basin_prob.min	60.6	dg.min.basin_prob.min	57.1
dca.angle.sd	61.6	dg.near.costs_runtime	61.7	dca.dist_ctr2best.sd	61.3	dca.costs_runtime	57.5
dca.dist_ctr2best.mean	62.6	dg.mean.best_attr.prob	63.1	dca.dist_ctr2best.mean	61.5	dg.near.basin_prob.min	61.2
dg near costs runtime	64.5 64.6	dg min basin prob min	64.5	dg near basin prob min	64.1	dg near basin prob max	61.5 61.0
dg.min.basin prob.min	65.4	dca.costs runtime	65.6	dg.min.best attr.prob	66.6	dg.mean.basin_prob.max	63.7
dg.mean.best_attr.prob	65.7	dg.mean.basin_prob.median	65.7	dg.mean.basin_prob.max	66.6	dg.mean.best_attr.prob	64.2
dg.min.best_attr.prob	66.2	dg.min.best_attr.prob	66.7	dca.costs_runtime	68.2	dg.min.best_attr.prob	64.4
dg.min.basin_prob.max	69.3	dg.near.basin_prob.max	69.1	dg.near.basin_prob.median	68.9	dg.min.basin_prob.max	65.5
dca.costs_runtime	70.1	dg.min.basin_prob.max	70.1	dg.min.basin_prob.max	69.2	dg.near.best_attr.prob	72.4
dg near best attr prob	71.2	dg near best attr prob	72.5	dg near basin certain mean	70.5	dg min basin uncertain min	76.2
dg.mean.basin prob.mean	72.1	dg.near.basin prob.min	73.0	dg.mean.best attr.prob	75.1	dg.min.basin_prob.median	78.5
dg.mean.basin_uncertain.min	72.5	dcc.convex.soft	73.7	dcc.concave.soft	75.6	dg.mean.basin_prob.median	78.7
dg.mean.basin_uncertain.median	72.8	dg.mean.basin_prob.mean	74.7	dcc.convex.soft	76.6	dg.near.basin_certain.mean	79.7
dg.near.basin_prob.max	73.7	dg.mean.basin_uncertain.median	76.6	dg.near.uncertain	77.9	dg.near.basin_prob.median	81.4
dcc.convex.soft	78.9	dg.mean.basin_uncertain.mean	76.7	dg.min.basin_prob.median	78.3	dg.min.costs_runtime	82.1
dg mean attractors	80.2	dg near basin prob median	79.9 82.4	dg mean basin prob median	70.0 83.1	dcc concave soft	823
dcc.concave.soft	80.7	dg.near.basin_certain.mean	83.4	dg.min.basin_prob.mean	83.2	dg.near.basin_uncertain.min	82.4
dg.min.basin_prob.median	81.0	dg.mean.costs_runtime	84.7	dg.mean.basin_prob.mean	84.3	dg.near.uncertain	82.9
dg.min.costs_runtime	81.8	dg.near.basin_certain.sum	85.4	dg.min.basin_uncertain.min	84.4	dcc.concave.hard	83.3
dg.mean.costs_runtime	84.0	dg.min.basin_uncertain.mean	86.2	dg.near.basin_uncertain.min	85.0	dg.mean.basin_prob.mean	83.7
dg.min.uncertain	84.5	dg.mean.basin_uncertain.min	86.2	dg.mean.basin_uncertain.mean	85.4	dg.near.basin_uncertain.max	84.2
dg min basin uncertain mean	87 0	dg min basin prob mean	87.4	dcc concave hard	86.8	dcc convex soft	85.6
dg.mean.basin_certain.sum	89.8	dg.mean.basin_certain.sum	88.9	dg.min.basin_uncertain.mean	87.1	dg.min.basin_uncertain.mean	86.3
dg.mean.uncertain	90.3	dg.near.uncertain	89.0	dg.near.basin_certain.max	87.9	dg.min.uncertain	87.5
dg.min.basin_prob.mean	90.4	dg.near.basin_certain.max	89.2	dg.mean.basin_uncertain.median	88.2	dg.near.basin_uncertain.median	88.0
dg.near.basin_uncertain.max	90.5	dg.min.basin_uncertain.median	90.2	dg.near.basin_uncertain.median	88.5	dg.near.basin_certain.median	88.0
dg min basin uncertain median	92.7	dg mean uncertain	91.2	dg mean basin uncertain min	91.1	dg mean basin uncertain max	90.4
dg.near.basin_certain.mean	93.1	dg.mean.basin_certain.mean	91.5	dg.near.basin_certain.median	93.0	dg.min.basin_uncertain.median	91.4
dg.mean.basin_uncertain.max	93.4	dg.min.costs_runtime	92.2	dg.min.basin_certain.mean	93.1	dg.mean.basin_uncertain.median	92.3
dg.mean.basin_certain.mean	93.7	dg.min.basin_certain.sum	93.0	dg.min.uncertain	94.4	dg.mean.basin_certain.sum	94.7
dg.mean.basin_certain.median	94.1	dg.mean.basin_certain.median	94.4	dg.min.basin_uncertain.max	95.2	dg.mean.uncertain	96.9
dg min basin certain sum	94.5	dg near basin uncertain max	90.8	dg min basin certain median	90.5	dø near basin certain min	90.9 97 1
dg.min.basin_uncertain.max	96.0	dg.min.basin_uncertain.max	97.6	dg.mean.tcells	98.3	dg.mean.tcells	97.6
dg.mean.tcells	98.5	dcc.concave.hard	98.3	dg.mean.costs_runtime	98.7	dg.mean.basin_certain.mean	98.0
dg.min.basin_certain.mean	99.2	dcc.convex.hard	98.7	dg.mean.uncertain	98.9	dg.min.basin_certain.sum	98.9
dg.near.basin_certain.median	100.5	dg.mean.basin_certain.max	98.8	dg.min.costs_runtime	99.8	dg.mean.basin_uncertain.sum	99.9
dg near basin certain sum	100.9	dg min basin certain median	99.5 100 5	dg mean basin certain sum	100.1	dg min tcells	100.1
dg.near.basin_uncertain.median	101.8	dg.mean.tcells	101.7	dg.near.pcells	103.8	dg.mean.basin_uncertain.min	100.7
dcc.concave.hard	103.0	dg.min.uncertain	102.1	dg.min.tcells	103.9	dg.min.basin_certain.mean	100.8
dg.min.basin_certain.max	103.5	dg.near.basin_certain.median	104.0	dg.near.basin_prob.mean	103.9	dg.min.basin_uncertain.sum	100.8
dg.min.basin_certain.median	105.1	dg.near.basin_uncertain.min	104.7	dg.near.basin_certain.min	104.5	dg.near.basin_certain.max	100.8
dg min tcells	105.5	dg mean basin uncertain sum	105.2	dg near tcells	104.8	dg near attractors	104.5
dg.near.basin_certain.max	106.0	dg.mean.pcells	106.2	dg.mean.basin_certain.mean	105.6	dg.mean.basin_certain.max	104.9
dg.mean.basin_uncertain.sum	107.2	dg.near.basin_uncertain.median	107.0	dg.near.attractors	106.2	dg.near.pcells	107.4
dg.min.basin_uncertain.sum	108.0	dg.mean.attractors	107.2	dcc.convex.hard	106.9	dcc.convex.hard	107.5
dg.near.basin_uncertain.mean	108.1	dg.min.tcells	109.0	dg.near.basin_uncertain.mean	106.9	dg.near.tcells	107.6
dg.near.basin_prob.mean	108.3	dg.mean.basin_certain.min	109.2	dg.mean.basin_certain.median	107.5	dg.min.basin_certain.median	108.0
dg near tcells	108.5	dg near basin certain min	112.5	dg mean basin uncertain sum	111.3	dg near best attr no	108.2
dg.near.pcells	109.9	dg.near.basin_uncertain.mean	113.4	dg.min.basin_uncertain.sum	111.7	dg.near.basin_prob.mean	110.7
dg.mean.basin_certain.min	110.1	dg.near.tcells	114.2	dg.min.basin_certain.min	111.8	dg.mean.basin_certain.min	110.7
dcc.convex.hard	113.5	dg.near.basin_prob.mean	114.7	dg.mean.attractors	115.0	dg.min.attractors	112.3
dg.min.basin_certain.min	117.3	dg.near.pcells	114.7	dg.near.best_attr.no	116.9	ag.min.pcells	114.3
dg min attractors	120.5	dg min pcells	115.4	dg min pcells	117.1	dg mean attractors	120.3
dg.min.pcells	120.7	dg.near.best_attr.no	118.8	dg.mean.pcells	118.3	dg.mean.pcells	120.5
dg.near.best_attr.no	124.7	dg.min.attractors	119.1	dg.mean.basin_certain.min	118.6	·	

Table S.6: Average rankings of the d_cm_angle (dca), d_cm_conv (dcc), d_cm_grad (dcg), and d_gcm (dg) features in C7-D4 for n = 640.

(a) *n* = 640

Feature	Rank
dca.y_ratio_best2worst.sd	43.2
dca.angle.mean	44.7
dcg.mean	47.1 50.5
dg.near.costs_runtime	50.8
dca.angle.sd	51.1
dca.dist_ctr2worst.mean	51.4 52.2
dca.y_ratio_best2worst.mean	52.6
dca.dist_ctr2best.sd	52.9
dcg.costs_runtime dca dist_ctr2worst_sd	54.9 55.4
dg.min.basin_prob.max	57.6
dg.near.basin_prob.min	62.2
dg.min.basin_prob.min dca_costs_runtime	62.3 62.9
dg.min.best_attr.prob	63.8
dg.min.basin_prob.median	64.3
dg.mean.basin_prob.min dg.near.best.attr.prob	64.9 65.7
dg.mean.basin_prob.max	67.8
dg.near.basin_prob.max	69.4
dg.mean.best_attr.prob	70.2
dg.near.basin_uncertain.min	75.1
dg.near.basin_prob.median	75.7
dg.min.basin_uncertain.mean	76.1
dcc.convex.soft	79.0 79.6
dg.min.uncertain	81.1
dg.mean.costs_runtime	81.7
dcc concave soft	83.5 84 1
dg.mean.basin_prob.median	84.2
dg.min.costs_runtime	85.9
dg.near.basin_uncertain.median	86.3
dg.min.basin_certain.mean	87.1
dg.min.basin_uncertain.min	88.1
dg.mean.basin_prob.mean	88.8 89.7
dg.min.basin_uncertain.max	91.0
dg.min.basin_certain.sum	92.3
dg.mean.basin_uncertain.median	92.4 93 3
dg.mean.uncertain	94.8
dg.mean.basin_uncertain.max	96.0
dg.min.basin_uncertain.sum	96.4 96.7
dg.near.uncertain	96.7
dcc.concave.hard	96.9
dg.mean.basin_certain.mean	97.9 98.7
dg.near.basin_certain.sum	99.2
dg.mean.basin_uncertain.sum	99.3
dg.near.attractors	100.9
dg.mean.basin_uncertain.min	101.2
dg.mean.basin_certain.median	101.7
dg.near.basin_prob.mean	104.0
dg.near.basin_certain.median	104.7
dg.min.pcells	104.8
dg.near.basin_uncertain.mean	105.0
dg.near.pcells	105.5
dg.mean.basin_certain.sum	106.2
dg.min.basin_certain.min dg near basin certain may	106.7
dg.mean.tcells	109.3
dg.mean.basin_certain.max	109.4
dg near basin certain min	111.6 113.0
dg.mean.basin_certain.min	114.1
dg.mean.pcells	118.1
ug.mean.attractors	118.8

Table S.7: Average accura	cy of C7-D2 with different	m for $n \in \mathbb{R}$	{3, 5, 1	10, 20, 40, 80	0, 160, 320, 640	}
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	Table S.7: Average accuracy of C7-D2 with different <i>m</i> for $n \in \{3, 5, 10, 20, 40, 80, 160, 320, 640\}$.										
		(a) M	Iultimodality	·			-	(b) G	lobal structure		
	m = 1	<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5	<i>m</i> = 10		m = 1	m = 2	<i>m</i> = 3	<i>m</i> = 5	<i>m</i> = 10
5	0.611 (0.422)	0.622 (0.415)	0.650 (0.432)	Na	Na	5	0.769 (0.395)	0.761 (0.412)	0.764 (0.402)	Na	Na
10	0.583 (0.439)	0.647 (0.434)	0.628 (0.425)	0.572 (0.415)	Na	10	0.700 (0.437)	0.692 (0.435)	0.703 (0.435)	0.686 (0.451)	Na
20	0.617 (0.415)	0.639 (0.413)	0.653 (0.425)	0.647 (0.433)	0.631 (0.430)	20	0.717 (0.421)	0.697 (0.440)	0.714 (0.425)	0.717 (0.429)	0.728 (0.421)
40	0.603 (0.448)	0.617 (0.441)	0.589 (0.434)	0.589 (0.438)	0.600 (0.429)	40	0.711 (0.460)	0.711 (0.460)	0.714 (0.456)	0.714 (0.456)	0.711 (0.460)
80	0.625 (0.455)	0.622 (0.454)	0.625 (0.451)	0.631 (0.459)	0.631 (0.444)	80	0.694 (0.452)	0.697 (0.453)	0.697 (0.453)	0.694 (0.453)	0.697 (0.453)
160	0.544(0.477)	0.561(0.475)	0.556(0.477)	0.592(0.472)	0.547(0.474)	160	0.622(0.471)	0.631(0.463)	0.633(0.462)	0.628 (0.469)	0.628(0.469)
520 640	0.575(0.482)	0.578 (0.472)	0.380(0.471) 0.581(0.488)	0.392(0.472) 0.583(0.488)	0.372(0.474) 0.575(0.483)	520 640	0.004(0.480) 0.672(0.474)	0.007 (0.462) 0.678 (0.468)	0.004(0.480) 0.675(0.471)	0.004(0.480) 0.675(0.471)	0.601(0.478)
040	0.580 (0.478)	0.363 (0.467)	0.301 (0.400)	0.303 (0.400)	0.575 (0.465)	040	0.072 (0.474)	0.078 (0.408)	0.075 (0.471)	0.075 (0.471)	0.075 (0.471)
		(c)	Separability					(d) V	ariable scaling		
	m = 1	m = 2	<i>m</i> = 3	<i>m</i> = 5	<i>m</i> = 10		m = 1	m = 2	<i>m</i> = 3	<i>m</i> = 5	m = 10
5	0.783 (0.374)	0.761 (0.421)	0.764 (0.400)	Na	Na	5	0.586 (0.475)	0.594 (0.480)	0.592 (0.478)	Na	Na
10	0.761 (0.408)	0.753 (0.411)	0.750 (0.415)	0.758 (0.408)	Na	10	0.564 (0.492)	0.572 (0.496)	0.556 (0.490)	0.528 (0.494)	Na
20	0.736 (0.402)	0.744 (0.401)	0.750 (0.403)	0.728 (0.425)	0.717 (0.433)	20	0.578 (0.499)	0.583 (0.504)	0.583 (0.504)	0.575 (0.498)	0.542 (0.509)
40	0.758 (0.400)	0.758 (0.401)	0.758 (0.398)	0.756 (0.399)	0.750 (0.404)	40	0.542 (0.499)	0.556 (0.492)	0.550 (0.494)	0.550 (0.494)	0.544 (0.497)
80	0.681 (0.454)	0.686 (0.447)	0.683 (0.455)	0.683 (0.452)	0.689 (0.449)	80	0.539 (0.507)	0.581 (0.501)	0.583 (0.504)	0.567 (0.496)	0.583 (0.504)
160	0.669(0.472)	0.667 (0.476)	0.672 (0.468)	0.667 (0.476)	0.669 (0.472)	160	0.567 (0.496)	0.583 (0.504)	0.583 (0.504)	0.583 (0.504)	0.583 (0.504)
520 640	0.672(0.474) 0.694(0.419)	0.669 (0.467)	0.669(0.472) 0.658(0.460)	0.661(0.472) 0.775(0.390)	0.672(0.468) 0.781(0.400)	520 640	$0.547 (0.495) \\ 0.542 (0.509)$	0.583(0.504) 0.542(0.509)	0.583(0.504) 0.542(0.509)	0.583(0.504) 0.542(0.509)	0.585(0.504) 0.569(0.496)
040	0.074 (0.417)	0.007 (0.455)	0.058 (0.400)	0.775 (0.570)	0.781 (0.400)	040	0.342 (0.307)	0.342 (0.307)	0.342 (0.307)	0.342 (0.307)	0.307 (0.470)
		(e) I	lomogeneity					(f) Basin size		
	m = 1	<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5	<i>m</i> = 10		m = 1	<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5	<i>m</i> = 10
5	0.669 (0.418)	0.647 (0.408)	0.664 (0.410)	Na	Na	5	0.525 (0.386)	0.522 (0.373)	0.517 (0.375)	Na	Na
10	0.767 (0.341)	0.769 (0.338)	0.758 (0.345)	0.758 (0.350)	Na	10	0.603 (0.376)	0.636 (0.384)	0.642 (0.365)	0.581 (0.372)	Na
20	0.775 (0.376)	0.731 (0.403)	0.731 (0.396)	0.717 (0.424)	0.736 (0.402)	20	0.536 (0.404)	0.558 (0.409)	0.569 (0.403)	0.581 (0.405)	0.533 (0.421)
40	0.811 (0.325)	0.744 (0.407)	0.744 (0.413)	0.739 (0.412)	0.731 (0.414)	40	0.586 (0.431)	0.614 (0.423)	0.608 (0.421)	0.619 (0.409)	0.608 (0.424)
80	0.717(0.431)	0.742(0.412)	0.717(0.419)	0.678 (0.448)	0.675(0.452)	80	0.594 (0.466)	0.597 (0.460)	0.600 (0.465)	0.583(0.4/1)	0.583 (0.467)
320	0.078(0.444) 0.742(0.438)	0.083(0.439) 0.761(0.417)	0.009(0.430) 0.747(0.441)	0.073(0.439)	0.030(0.434) 0.739(0.438)	320	0.011(0.430) 0.497(0.461)	0.014(0.447) 0.508(0.446)	0.528 (0.452)	0.028(0.444) 0.511(0.454)	0.019(0.439) 0.500(0.456)
640	0.742 (0.450)	0.781 (0.411)	0.742 (0.423)	0.739 (0.438)	0.781 (0.410)	640	0.511 (0.468)	0.528 (0.471)	0.511 (0.467)	0.522 (0.474)	0.517 (0.474)
		(11)	(111)	((,		(111)	(,	(111)	(,	(,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
		(g)	GL contrast					(h) O	verall average		
	m = 1	<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5	<i>m</i> = 10		m = 1	m = 2	<i>m</i> = 3	<i>m</i> = 5	m = 10
5	0.536 (0.414)	0.536 (0.411)	0.522 (0.426)	Na	Na	5	0.640 (0.417)	0.635 (0.420)	0.639 (0.422)	Na	Na
10	0.600 (0.413)	0.619 (0.410)	0.619 (0.405)	0.617 (0.404)	Na	10	0.654 (0.418)	0.670 (0.415)	0.665 (0.412)	0.643 (0.417)	Na
20	0.628 (0.383)	0.617 (0.384)	0.636 (0.394)	0.600 (0.407)	0.619 (0.408)	20	0.655 (0.417)	0.653 (0.421)	0.662 (0.420)	0.652 (0.429)	0.644 (0.433)
40	0.583(0.430)	0.597 (0.428)	0.578 (0.433)	0.594(0.436)	0.589 (0.439)	40	0.656(0.434)	0.657 (0.436)	0.649 (0.436)	0.652(0.435)	0.648(0.438)
60 160	0.622(0.459)	0.619(0.458)	0.017 (0.400)	0.608 (0.457)	0.392 (0.447)	ðU 140	0.039 (0.450)	0.649(0.451) 0.623(0.458)	0.640 (0.453)	0.635 (0.457)	0.615 (0.454)
320	0.031(0.446) 0.653(0.459)	0.022 (0.445) 0.647 (0.436)	0.008 (0.435)	0.008 (0.442) 0.639 (0.440)	0.000(0.444) 0.639(0.440)	320	0.017 (0.400) 0.621 (0.468)	0.623 (0.438)	0.020(0.438) 0.629(0.461)	0.628(0.460)	0.013(0.439) 0.624(0.463)
640	0.589 (0.492)	0.614(0.473)	0.633 (0.473)	0.631 (0.482)	0.594 (0.481)	640	0.626 (0.466)	0.628 (0.467)	0.620 (0.468)	0.638 (0.466)	0.642 (0.463)
	(((111)()	(((((((

	(a) N	Iultimodality			(b) G	lobal structure	
	<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5		<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5
5	0.522 (0.423)	0.550 (0.436)	Na	5	0.778 (0.400)	0.772 (0.408)	Na
10	0.561 (0.414)	0.558 (0.425)	0.544 (0.419)	10	0.719 (0.426)	0.728 (0.412)	0.728 (0.424)
20	0.550 (0.436)	0.525 (0.434)	0.536 (0.419)	20	0.733 (0.416)	0.733 (0.412)	0.733 (0.413)
40	0.564 (0.449)	0.567 (0.456)	0.575 (0.450)	40	0.714 (0.456)	0.717 (0.448)	0.722 (0.447)
80	0.544 (0.436)	0.567 (0.442)	0.578 (0.440)	80	0.706 (0.440)	0.708 (0.441)	0.714 (0.437)
160	0.528 (0.445)	0.533 (0.450)	0.547 (0.439)	160	0.656 (0.453)	0.661 (0.453)	0.667 (0.448)
320	0.542 (0.472)	0.553 (0.474)	0.542 (0.473)	320	0.664 (0.474)	0.661 (0.478)	0.669 (0.478)
640	0.536 (0.466)	0.528 (0.462)	0.506 (0.468)	640	0.683 (0.461)	0.683 (0.461)	0.678 (0.464)

Table S.8: Average accuracy of C7-D4 with different *m* for $n \in \{5, 10, 20, 40, 80, 160, 320, 640\}$.

(c)	Separability		
2	m - 3	m = 5	_

	m = 2	<i>m</i> = 3	<i>m</i> = 5		<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5
5	0.761 (0.409)	0.769 (0.417)	Na	5	0.636 (0.469)	0.633 (0.464)	Na
10	0.758 (0.414)	0.767 (0.413)	0.772 (0.413)	10	0.528 (0.486)	0.536 (0.482)	0.542 (0.478)
20	0.731 (0.410)	0.739 (0.408)	0.722 (0.414)	20	0.536 (0.504)	0.531 (0.501)	0.539 (0.507)
40	0.775 (0.402)	0.778 (0.399)	0.769 (0.395)	40	0.533 (0.490)	0.550 (0.497)	0.553 (0.496)
80	0.719 (0.425)	0.722 (0.423)	0.711 (0.431)	80	0.581 (0.501)	0.583 (0.504)	0.581 (0.501)
160	0.708(0.440)	0.719 (0.436)	0.725 (0.436)	160	0.583 (0.504)	0.583 (0.504)	0.583 (0.504)
320	0.731 (0.433)	0.733 (0.431)	0.725 (0.432)	320	0.544 (0.506)	0.547 (0.504)	0.558 (0.497)
640	0.744 (0.413)	0.739 (0.419)	0.753 (0.412)	640	0.542 (0.509)	0.542 (0.509)	0.542 (0.509)

(d) Variable scaling

	(e) I	Iomogeneity			(f) Basin size	
	<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5		<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5
5	0.636 (0.412)	0.636 (0.421)	Na	5	0.478 (0.403)	0.489 (0.403)	Na
10	0.706 (0.395)	0.697 (0.397)	0.694 (0.410)	10	0.531 (0.388)	0.506 (0.401)	0.508 (0.403)
20	0.769 (0.367)	0.775 (0.359)	0.764 (0.360)	20	0.483 (0.425)	0.481 (0.432)	0.497 (0.437)
40	0.781 (0.368)	0.786 (0.361)	0.789 (0.356)	40	0.533 (0.440)	0.528 (0.441)	0.525 (0.444)
80	0.706 (0.434)	0.733 (0.414)	0.742 (0.418)	80	0.519 (0.448)	0.528 (0.457)	0.508 (0.471)
160	0.656(0.465)	0.667 (0.449)	0.678 (0.444)	160	0.472 (0.459)	0.475 (0.454)	0.483 (0.457)
320	0.772 (0.408)	0.778 (0.410)	0.761 (0.410)	320	0.400(0.478)	0.403 (0.476)	0.406 (0.476)
640	0.758 (0.424)	0.758 (0.424)	0.747 (0.435)	640	0.492 (0.475)	0.483 (0.479)	0.492 (0.477)

	(g)	GL contrast			(h) 0	verall average	
	<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5		<i>m</i> = 2	<i>m</i> = 3	<i>m</i> = 5
5	0.503 (0.407)	0.508 (0.408)	Na	5	0.616 (0.426)	0.623 (0.429)	Na
10	0.592 (0.419)	0.569 (0.398)	0.561 (0.409)	10	0.628 (0.423)	0.623 (0.423)	0.621 (0.427)
20	0.558 (0.410)	0.544 (0.410)	0.542 (0.412)	20	0.623 (0.432)	0.618 (0.432)	0.619 (0.431)
40	0.522 (0.417)	0.508 (0.416)	0.508 (0.417)	40	0.632 (0.440)	0.633 (0.440)	0.635 (0.438)
80	0.553 (0.446)	0.567 (0.444)	0.558 (0.443)	80	0.618 (0.447)	0.630 (0.447)	0.627 (0.450)
160	0.547 (0.440)	0.536 (0.443)	0.547 (0.444)	160	0.593 (0.457)	0.596 (0.455)	0.604 (0.453)
320	0.550 (0.473)	0.558 (0.474)	0.550 (0.478)	320	0.600 (0.471)	0.605 (0.471)	0.602 (0.470)
640	0.544(0.486)	0.561 (0.480)	0.561 (0.478)	640	0.614 (0.466)	0.613 (0.466)	0.611 (0.467)



S.5: Figure **Boxplots** of values of ela_meta.lin_simple.adj_r2 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with *n* = 160.



(a) ela_meta.lin_simple.intercept (b) d_ela_meta.lin_simple.intercept

Figure S.6: Boxplots of values of ela_meta.lin_simple.intercept and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with *n* = 160.



(a) ela_meta.lin_simple.coef.min (b) d_ela_meta.lin_simple.coef.min

Figure **S.7**: Boxplots of values of ela_meta.lin_simple.coef.min and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with *n* = 160.



(a) ela_meta.lin_simple.coef.max (b) d_ela_meta.lin_simple.coef.max

S.8: Figure **Boxplots** of values of ela_meta.lin_simple.coef.max and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with *n* = 160.



(a) (b)
ela_meta.lin_simple.coef.max_by_mind_ela_meta.lin_simple.coef.max_by_min

S.9: **Boxplots** of of Figure values ela_meta.lin_simple.coef.max_by_min and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 160.



Figure **Boxplots** S.10: of values of ela_meta.lin_w_interact.adj_r2 and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 160.



Figure S.11: Boxplots of values of ela_meta.quad_simple.adj_r2 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.12: Boxplots of values of ela_meta.quad_simple.cond and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



 $\begin{array}{c} (a) \texttt{ela_meta.quad_w_interact.adj_r2} (b) \\ \texttt{d_ela_meta.quad_w_interact.adj_r2} \end{array}$

Figure S.13: Boxplots of values of ela_meta.quad_w_interact.adj_r2 and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 160.



Figure S.14: Boxplots of values of ela_meta.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.15: Boxplots of values of ela_meta.costs_runtime and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.16: Boxplots of values of ela_level.mmce_lda_10 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.17: Boxplots of values of ela_level.mmce_qda_10 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.18: Boxplots of values of ela_level.mmce_mda_10 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.19: Boxplots of values of ela_level.lda_qda_10 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.20: Boxplots of values of ela_level.lda_mda_10 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.21: Boxplots of values of ela_level.qda_mda_10 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.22: Boxplots of values of ela_level.mmce_lda_25 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.23: Boxplots of values of ela_level.mmce_qda_25 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.24: Boxplots of values of ela_level.mmce_mda_25 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.25: Boxplots of values of ela_level.lda_qda_25 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.26: Boxplots of values of ela_level.lda_mda_25 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.27: Boxplots of values of ela_level.qda_mda_25 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.28: Boxplots of values of ela_level.mmce_lda_50 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.29: Boxplots of values of ela_level.mmce_qda_50 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.30: Boxplots of values of ela_level.mmce_mda_50 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.31: Boxplots of values of ela_level.lda_qda_50 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.32: Boxplots of values of ela_level.lda_mda_50 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.33: Boxplots of values of ela_level.qda_mda_50 and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.34: Boxplots of values of ela_level.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 160.



Figure S.35: Boxplots of values of ela_level.costs_runtime and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 160.



Figure S.36: Boxplots of values of cm_grad.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.37: Boxplots of values of cm_grad.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.38: Boxplots of values of cm_grad.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.39: Boxplots of values of cm_grad.costs_runtime and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb| cm_angle.dist_ctr2best.mean (b) d_cm_angle.dist_ctr2best.mean \\$

Figure S.40: Boxplots of values of cm_angle.dist_ctr2best.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.41: Boxplots of values of cm_angle.dist_ctr2best.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb| cm_angle.dist_ctr2worst.mean (b) d_cm_angle.dist_ctr2worst.mean \\$

Figure S.42: Boxplots of values of cm_angle.dist_ctr2worst.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb"cm_angle.dist_ctr2worst.sd" (b) \verb"d_cm_angle.dist_ctr2worst.sd" (b) \verb"d_cm_angle.dist_ctr2worst.sd" (b) \verb"d_cm_angle.dist_ctr2worst.sd" (c) "d_cm_angle.dist_ctr2worst.sd" (c) "$

Figure S.43: Boxplots of values of cm_angle.dist_ctr2worst.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.44: Boxplots of values of cm_angle.angle.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.45: Boxplots of values of cm_angle.angle.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



d_cm_angle.y_ratio_best2worst.mean

Figure S.46: Boxplots of values of cm_angle.y_ratio_best2worst.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



d_cm_angle.y_ratio_best2worst.sd

Figure S.47: Boxplots of values of cm_angle.y_ratio_best2worst.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.48: Boxplots of values of cm_angle.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.49: Boxplots of values of cm_angle.costs_runtime and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.50: Boxplots of values of cm_conv.convex.hard and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.51: Boxplots of values of cm_conv.concave.hard and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.52: Boxplots of values of cm_conv.convex.soft and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.53: Boxplots of values of cm_conv.concave.soft and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.54: Boxplots of values of cm_conv.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.55: Boxplots of values of cm_conv.costs_runtime and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.56: Boxplots of values of gcm.min.attractors and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.57: Boxplots of values of gcm.min.pcells and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.58: Boxplots of values of gcm.min.tcells and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.59: Boxplots of values of gcm.min.uncertain and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.60: Boxplots of values of gcm.min.basin_prob.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.61: Boxplots of values of gcm.min.basin_prob.mean and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.62: Boxplots of values of gcm.min.basin_prob.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.63: Boxplots of values of gcm.min.basin_prob.max and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.64: Boxplots of values of gcm.min.basin_prob.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.65: Boxplots of values of gcm.min.basin_certain.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.66: Boxplots of values of gcm.min.basin_certain.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb"gcm.min.basin_certain.median" (b) d_gcm.min.basin_certain.median"$

Figure S.67: Boxplots of values of gcm.min.basin_certain.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.68: Boxplots of values of gcm.min.basin_certain.max and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.69: Boxplots of values of gcm.min.basin_certain.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.70: Boxplots of values of gcm.min.basin_certain.sum and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.71: Boxplots of values of gcm.min.basin_uncertain.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.min.basin_uncertain.mean (b) d_gcm.min.basin_uncertain.mean

Figure S.72: Boxplots of values of gcm.min.basin_uncertain.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $\begin{array}{ll} \text{(a) gcm.min.basin_uncertain.median (b)} \\ & & \text{d_gcm.min.basin_uncertain.median} \end{array}$

Figure S.73: Boxplots of values of gcm.min.basin_uncertain.median and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.74: Boxplots of values of gcm.min.basin_uncertain.max and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.75: Boxplots of values of gcm.min.basin_uncertain.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb"gcm.min.basin_uncertain.sum" (b) \verb"d_gcm.min.basin_uncertain.sum" (b) \verb"d_gcm.min.basin_uncertain.sum" (b) \verb"d_gcm.min.basin_uncertain.sum" (c) \verb"d"gcm.min.basin_uncertain.sum" (c) "d"gcm.min.basin_uncertain.sum" (c) "d"gcm.min.basin_uncer$

Figure S.76: Boxplots of values of gcm.min.basin_uncertain.sum and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.77: Boxplots of values of gcm.min.best_attr.prob and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.78: Boxplots of values of gcm.min.best_attr.no and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.79: Boxplots of values of gcm.min.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.80: Boxplots of values of gcm.min.costs_runtime and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.81: Boxplots of values of gcm.mean.attractors and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.82: Boxplots of values of gcm.mean.pcells and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.83: Boxplots of values of gcm.mean.tcells and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.84: Boxplots of values of gcm.mean.uncertain and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.85: Boxplots of values of gcm.mean.basin_prob.min and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.86: Boxplots of values of gcm.mean.basin_prob.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.87: Boxplots of values of gcm.mean.basin_prob.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.88: Boxplots of values of gcm.mean.basin_prob.max and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.89: Boxplots of values of gcm.mean.basin_prob.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.90: Boxplots of values of gcm.mean.basin_certain.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.91: Boxplots of values of gcm.mean.basin_certain.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.mean.basin_certain.median (b) d_gcm.mean.basin_certain.median

Figure S.92: Boxplots of values of gcm.mean.basin_certain.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.mean.basin_certain.max (b) d_gcm.mean.basin_certain.max

Figure S.93: Boxplots of values of gcm.mean.basin_certain.max and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.94: Boxplots of values of gcm.mean.basin_certain.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.95: Boxplots of values of gcm.mean.basin_certain.sum and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.mean.basin_uncertain.min (b) d_gcm.mean.basin_uncertain.min

Figure S.96: Boxplots of values of gcm.mean.basin_uncertain.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb"gcm.mean.basin_uncertain.mean" (b) \verb"d_gcm.mean.basin_uncertain.mean" (b) \verb"d_gcm.mean.basin_uncertain.mean" (b) \verb"d_gcm.mean.basin_uncertain.mean" (c) \verb"d_gcm.mean.basin_uncertain.basin_uncertain.mean" (c) \verb"d_gcm.mean.basin_uncertain.b$

Figure S.97: Boxplots of values of gcm.mean.basin_uncertain.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a)gcm.mean.basin_uncertain.median(b)

d_gcm.mean.basin_uncertain.median

Figure S.98: Boxplots of values of gcm.mean.basin_uncertain.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.mean.basin_uncertain.max (b) d_gcm.mean.basin_uncertain.max

Figure S.99: Boxplots of values of gcm.mean.basin_uncertain.max and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb"gcm.mean.basin_uncertain.sd" (b) \verb"d_gcm.mean.basin_uncertain.sd"$

Figure S.100: Boxplots of values of gcm.mean.basin_uncertain.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.101: Boxplots of values of gcm.mean.basin_uncertain.sum and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20}

with n = 5.



Figure S.102: Boxplots of values of gcm.mean.best_attr.prob and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.103: Boxplots of values of gcm.mean.best_attr.no and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.104: Boxplots of values of gcm.mean.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.105: Boxplots of values of gcm.mean.costs_runtime and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.106: Boxplots of values of gcm.near.attractors and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.107: Boxplots of values of gcm.near.pcells and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.108: Boxplots of values of gcm.near.tcells and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.109: Boxplots of values of gcm.near.uncertain and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.110: Boxplots of values of gcm.near.basin_prob.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.111: Boxplots of values of gcm.near.basin_prob.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.near.basin_prob.median

(b) d_gcm.near.basin_prob.median

Figure S.112: Boxplots of values of gcm.near.basin_prob.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.113: Boxplots of values of gcm.near.basin_prob.max and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.114: Boxplots of values of gcm.near.basin_prob.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.115: Boxplots of values of gcm.near.basin_certain.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.116: Boxplots of values of gcm.near.basin_certain.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.near.basin_certain.median (b) d_gcm.near.basin_certain.median

Figure S.117: Boxplots of values of gcm.near.basin_certain.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.near.basin_certain.max (b) d_gcm.near.basin_certain.max

Figure S.118: Boxplots of values of gcm.near.basin_certain.max and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.119: Boxplots of values of gcm.near.basin_certain.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.120: Boxplots of values of gcm.near.basin_certain.sum and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



 $(a) \verb"gcm.near.basin_uncertain.min" (b) d_gcm.near.basin_uncertain.min"$

Figure S.121: Boxplots of values of gcm.near.basin_uncertain.min and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.near.basin_uncertain.mean (b) d_gcm.near.basin_uncertain.mean

Figure S.122: Boxplots of values of gcm.near.basin_uncertain.mean and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



d_gcm.near.basin_uncertain.median

Figure S.123: Boxplots of values of gcm.near.basin_uncertain.median and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.near.basin_uncertain.max (b) d_gcm.near.basin_uncertain.max

Figure S.124: Boxplots of values of gcm.near.basin_uncertain.max and its dimensionality reduction version on 15 instances of f_1, f_6, f_{10}, f_{15} , and f_{20} with n = 5.



Figure S.125: Boxplots of values of gcm.near.basin_uncertain.sd and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.near.basin_uncertain.sum (b) d_gcm.near.basin_uncertain.sum

Figure S.126: Boxplots of values of gcm.near.basin_uncertain.sum and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.127: Boxplots of values of gcm.near.best_attr.prob and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.128: Boxplots of values of gcm.near.best_attr.no and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



(a) gcm.near.costs_fun_evals (b) d_gcm.near.costs_fun_evals

Figure S.129: Boxplots of values of gcm.near.costs_fun_evals and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.130: Boxplots of values of gcm.near.costs_runtime and its dimensionality reduction version on 15 instances of f_1 , f_6 , f_{10} , f_{15} , and f_{20} with n = 5.



Figure S.131: Kendall τ values of ela_level and d_ela_level for n = 160.



Figure S.132: Kendall τ values of gcm and d_gcm for n = 5.



Figure S.133: Kendall τ values of cm_angle and d_cm_angle for n = 5.



Figure S.134: Kendall τ values of cm_conv and d_cm_conv for n=5.



Figure S.135: Kendall τ values of cm_grad and d_cm_grad for n = 5.