A Note on Multi-Funnel Functions for Expensive Optimization Scenario

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ABSTRACT

This paper presents an characteristic analysis of multi-funnel functions for expensive computational budgets. IPOP-CMA-ES is applied to three multi-funnel functions from the BBOB benchmarks for two different budget scenarios. The experimental analysis using Fitness Distance Correlation (FDC) shows that search spaces of IPOP-CMA-ES in multi-funnel functions significantly differ depending on the maximum number of fitness evaluations.

Categories and Subject Descriptors

G.1.6 [**Mathematics of Computing**]: Optimization—*Global optimization*; I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*

1. INTRODUCTION

Some real-world applications of Evolutionary Algorithms (EA) require executing a very expensive simulations (up to 10 minutes/run) in order to evaluate the fitness of a single individual. Thus, in recent years, there has been much research on such expensive optimization problems, where only a small computational budget can be used for the search.

The real parameter optimization benchmark problems for evaluating the search performance of EA for expensive optimization problem have recently been proposed. Typical benchmark suites are the BBOB benchmarks [3] for expensive scenario[1] and the CEC2014 expensive optimization benchmarks [5]. However, these benchmarks consist of the widely used functions and maximum number of fitness evaluations (MaxEvals) is merely set to small limited number (e.g. MaxEvals $= 10^2 \times D$, where D is the benchmark problem dimensionality). Thus, an suitability of these benchmarks might be open to question.

In this paper, we consider muti-funnel functions, which have a high deceptive structure and are considered as EA-hard problems, for expensive computational budget scenario. The previous work [6] showed that CMA-ES struggle to search the optimal solution for multi-funnel functions for *cheap scenario* where EA can utilize a relatively large computational budget. In contrast, for *expensive scenario* where only a small computational budget can be used for the search, EA might be not able to obtain the local optimal solution in a suboptimal basin and is not affected multi-funnel structure. Since the fraction of the search space that an EA can explore depends on the available computational budget, the problem characteristics might differ whether a budget scenario is an expensive scenario (e.g. MaxEvals $= 10^2 \times D$) or a cheap scenario (e.g. MaxEvals $= 10^4 \times D$). To clarify above issues, this paper presents an empirical analysis of multi-funnel functions for expensive/cheap computational budgets scenario using Fitness Distance Correlation (FDC) [4].

[1]http://coco.gforge.inria.fr/doku.php?id=bbob-2013

2. FITNESS DISTANCE CORRELATION

FDC [4] is a landscape analysis method which analyzes the correlation between $f(\boldsymbol{x})$, the fitness of a D-dimensional solution vector $\boldsymbol{x} = (x_1, ..., x_D)$ and $\mathrm{d}(\boldsymbol{x}, \boldsymbol{x}^*)$, the distance between $\boldsymbol{x}$ and the optimal solution vector $\boldsymbol{x}^*$. Euclidean distance is a general distance metric $\mathrm{d}(\cdot, \cdot)$ for real parameter optimization problem. The global structure (e.g. single-funnel or multi-funnel structure) of a given problem can be understood by analyzing the correlation between $f(\boldsymbol{x})$ and $d(\boldsymbol{x}, \boldsymbol{x}^*)$.

While the FDC can be visualized by using a 2-D plot of $f(\boldsymbol{x})$ (vertical axis) and $\mathrm{d}(\boldsymbol{x}, \boldsymbol{x}^*)$ (horizontal axis), as shown in Figure 1, following r_{FDC} metric [4] is used for a quantitative analysis: $r_{\mathrm{FDC}} = \frac{c_{FD}}{s_F s_D}, c_{FD} = \frac{1}{n}\sum_{i=1}^{n}(f_i - \bar{f})(d_i - \bar{d})$. Where n is the number of sampled solution vectors $\{\boldsymbol{x}_1, ..., \boldsymbol{x}_n\}$, f_i and d_i are the fitness and distance (from $\boldsymbol{x}^*$) of the i-th vector $\boldsymbol{x}_i$ $(1 \leq i \leq n)$, and $\bar{f}, \bar{d}, s_F, s_D$ are the means and standard deviations of fitness values and distances, respectively. As r_{FDC} approaches 1, the search space has a more pronounced big valley structure, which is an easy problem for EA. On the other hand, r_{FDC} closes to 0 indicates that the global structure is a weak or none, and a negative value of r_{FDC} indicates a deceptive structure.

3. EXPERIMENT RESULTS

To analyze characteristics of multi-funnel functions, we apply IPOP-CMA-ES[2] [1] to the 10-dimensional f_{21}, f_{22} and f_{24} from the BBOB benchmarks [3] for two different budged

[2]We conducted the same experiments for DE and PSO, but the obtained results are very similar to the IPOP-CMA-ES results shown in Figure 1.

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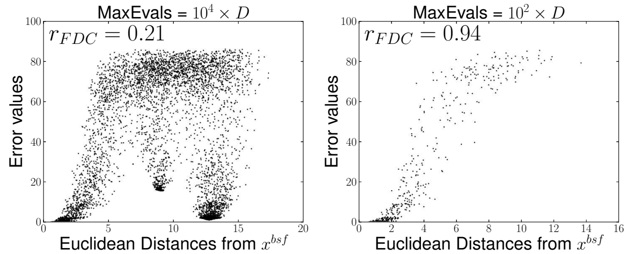
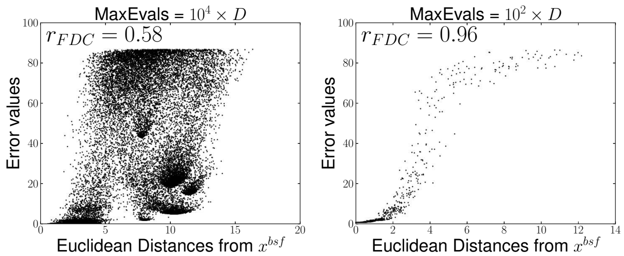
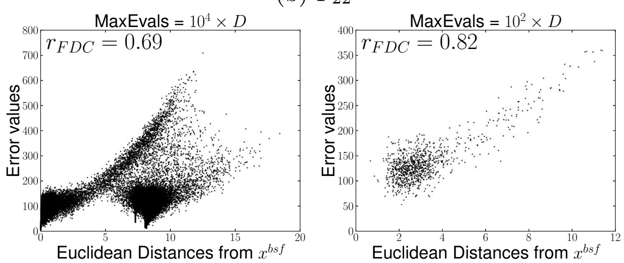


(a) f_{21}



(b) F_{22}



(c) f_{24}

Figure 1: Figures (a) – (c) show the FDC for the 10-dimensional f_{21}, f_{22} and f_{24} from the BBOB benchmarks respectively. Solutions generated by IPOP-CMA-ES and corresponding error values in the best run out of 25 runs are plotted. The median r_{FDC} values are also shown in the figures. The horizontal axis represents the Euclidean distance between $\boldsymbol{x}$ and the best-so-far solution $\boldsymbol{x}^{bsf}$, and the vertical axis represents the error value of $\boldsymbol{x}$ (lower is better). Left and right figures show the results for cheap scenario (MaxEvals= $10^4 \times D$) and expensive scenario (MaxEvals= $10^2 \times D$) respectively.

scenarios: expensive scenario (MaxEvals $= 10^2 \times D$) and cheap scenario (MaxEvals $= 10^4 \times D$). f_{21} and f_{22} are generated by using MSG function generator [2] and have 101 and 21 peaks respectively. f_{24} is designed by using double-funnel function generator proposed by Lunacek et al [6] and has two funnels. For IPOP-CMA-ES, we used the control parameter values that were suggested in [1]. On each problem, IPOP-CMA-ES is executed 25 times with same function instance.

Figure 1 shows the FDC scatter plots and the r_{FDC} values for IPOP-CMA-ES. Note that the best-so-far solution $\boldsymbol{x}^{bsf}$ is used for the FDC analysis instead of the optimal solution $\boldsymbol{x}^*$ since our interest is not problem itself such as traditional landscape analysis studies, but search spaces of EA. The shapes of their FDC scatter plots are significantly different depending on budget scenario. For cheap scenario, IPOP-CMA-ES can clearly find out the global structure. For instance, three funnels can be seen on the result of f_{21} for cheap scenario (Figure 1(a) left). In contrast, due to a small computational budget, IPOP-CMA-ES can only find a funnel on the result of f_{21} for expensive scenario. This result is also shown in f_{24} having the double-funnel landscape. While we can see clear two valleys of f_{24} for cheap scenario (Figure 1(c) left), only a valley can be seen for expensive scenario (Figure 1(c) right).

The r_{FDC} values for expensive scenario tend to be higher than those for cheap scenario. Especially, the r_{FDC} values in f_{21}, f_{22} for expensive scenario are 0.94 and 0.96 respectively. This means that f_{21} and f_{22} are almost the same as single funnel function having high fitness-distance correlation. This observation is significantly incompatible with the original function properties of f_{21} and f_{22}.

4. DISCUSSION AND CONCLUSION

In Section 3, we confirmed that the search spaces of IPOP-CMA-ES in multi-funnel functions significantly differ depending on the available computational budgets. This results demonstrate that the current BBOB benchmarks might be inappropriate for evaluating the performance of EA for solving expensive optimization problem. The search performance of IPOP-CMA-ES might be invariant whether a given function has the single-funnel/multi-funnel structure for expensive scenario. This misleads the compared results among the different algorithms. Let us consider evaluating method A and method B for a single-funnel function (e.g. Rastrigin function) and a multi-funnel function (e.g. Double-Rastrigin function) for expensive scenario. Due to above reason, he/she might incorrectly conclude as: “For expensive scenario, the method A perform better than the method B independent from the global structure”.

To avoid this incorrect conclusion, following two directions can be considered as our future works: (1) eliminating benchmark functions having unexpected r_{FDC} values or shapes such as f_{21}, f_{22} and f_{24} from a benchmark set, (2) designing new benchmark functions for expensive scenario.

Acknowledgments

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