On the Unbounded External Archive and Population Size in Preference-based EMO Using a Reference Point

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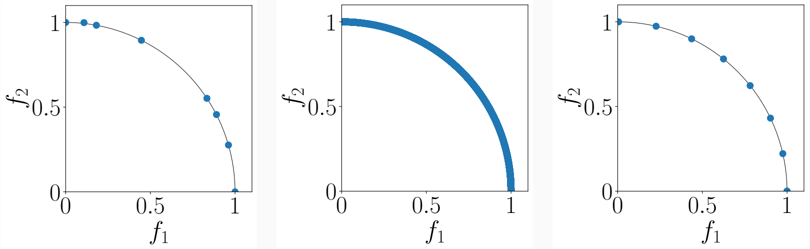
An unbounded archive maintains all non-dominated solutions found so far

The archive can be incorporated into any EMO algorithm

It can maintain all solutions potentially preferred by the DM

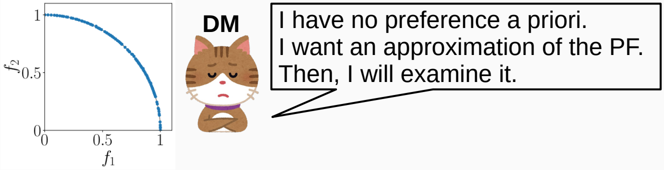
Final pop. of size 8 | Archive of size $4\,933$ | Its subset of size 8



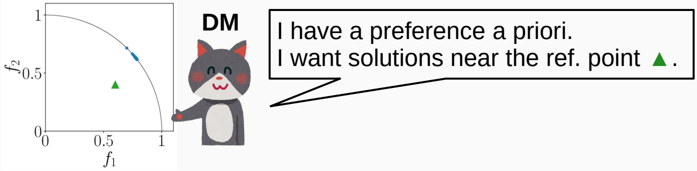
- The DM does not want to examine so many solutions
- A postprocessing method is available to select only k solutions

Preference-based EMO using a reference point (PBEMO) [Deb 06]

EMO (e.g., NSGA-II) approximates the whole Pareto front



PBEMO (e.g., R-NSGA-II) approximates a region of interest (ROI)



- Approximating the ROI can be easier than approximating the PF
- Showing only preferred solutions can reduce the DM's cognitive load

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Kalyanmoy Deb, J. Sundar, Udaya Bhaskara, and Shamik Chaudhuri: Reference Point Based Multi-Objective Optimization Using Evolutionary Algorithms. Int. J. Comput. Intell. 2, 3 (2006), 273–286.

Three main contributions of this work

Long-term goal: Establishing a benchmarking methodology for PBEMO

1. **It proposes a preference-based postprocessing method**
   - Existing pp methods cannot handle the DM's preference information
2. **It investigates effects of the unbounded archive in PBEMO**
   - Except for [Fonseca 93], no previous study used the archive
3. **It investigates the best population size for PBEMO**
   - Intuitively, PBEMO requires only a small population size
   - But, most previous studies used a large population size

Ref.	Population size
NSGA-II	100
R-NSGA-II	100 – 500
PBEA	20 – 200
R-MEAD2	200 – 350

Carlos M. Fonseca, Peter J. Fleming: Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization. ICGA 1993: 416-423

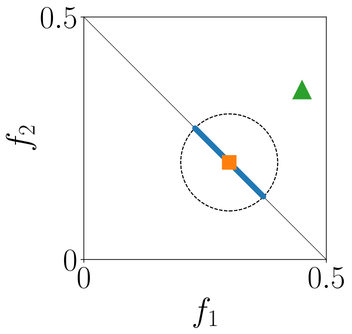
Preliminary: The region of interest (ROI) considered in this work

Just as there is no standard GA, there is no standard ROI [Tanabe 23]

- If there are 100 researchers, there are 100 different GAs and ROIs

ROI based on the Pareto opt. point ■ closest to the ref. point ▲

- A set of all Pareto opt. points in a sphere of radius r centered at ■



$$\blacksquare = \mathop{\arg\min}_{\bullet \in \mathrm{PF}} \{\mathtt{distance}(\bullet, \blacktriangle)\},$$

$$\mathrm{ROI} = \{\bullet \in \mathrm{PF} \mid \mathtt{distance}(\bullet, \blacksquare) < r\}.$$

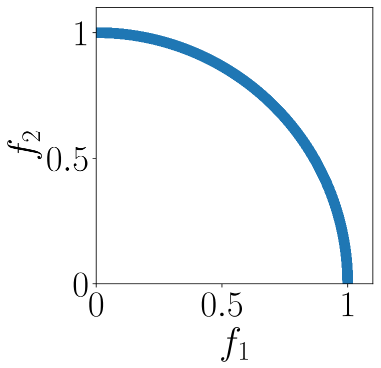
r is supplied by the DM or analyst

Ryoji Tanabe, Ke Li: Quality Indicators for Preference-based Evolutionary Multi-objective Optimization Using a Reference Point: A Review and Analysis. CoRR abs/2301.12148 (2023)

Proposed preference-based postprocessing method

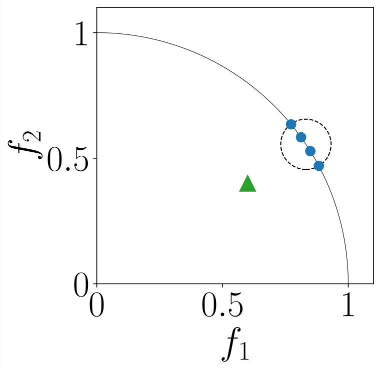
Input

Unbounded archive



Output

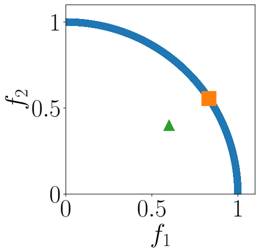
Its subset of size k



Difficulty: A set of k points should approximates the ROI

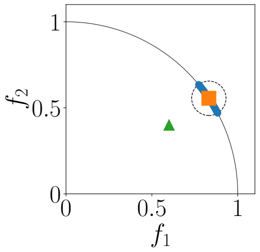
Proposed preference-based postprocessing method

1. **Find the closest point ■ to the ref. point ▲ from the archive**



$$■ = \arg\min_{\bullet \in \text{archive}} \{\texttt{distance}(\bullet, ▲)\}$$

2. **Select points in the region of a sphere of radius r centered at ■**



$$\mathcal{X} = \left\{\bullet \in \text{archive} \,\middle|\, \texttt{distance}\left(\bullet, ■\right) < r\right\}$$

There are three cases

3.1 $|\mathcal{X}| = k$

3.2 $|\mathcal{X}| < k$

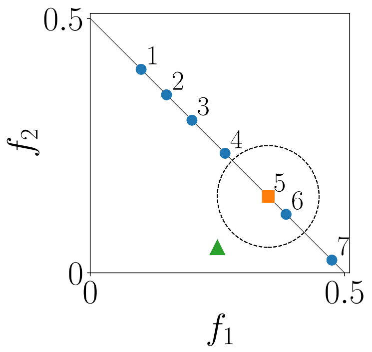
3.3 $|\mathcal{X}| > k$

3.2 The size of the subset $\mathcal{X}$ is less than k

The proposed method selects unselected points closest to ■

- The DM is interested in closer points to the center point ■
- even though they are out of the approximated ROI

Example: $k = 3$, $|\mathcal{X}| = 2$, **and** $|\textbf{archive}| = 7$



$\mathcal{X} = \{■^5, ●^6\}$, $|\mathcal{X}| < 3$

$●^4$ is closest to $■^5$

$●^4$ is added to $\mathcal{X}$

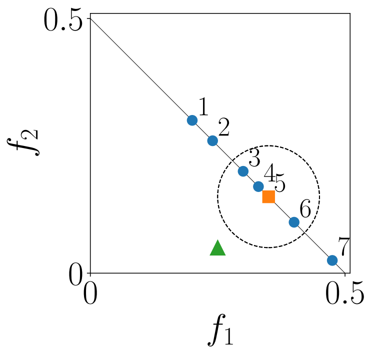
$\mathcal{X} = \{●^4, ■^5, ●^6\}$, $|\mathcal{X}| = 3$

3.3 The size of the subset $\mathcal{X}$ is greater than k

A conventional postprocessing method can be used

- It selects k representative points from $\mathcal{X}$
- Iterative distance-based subset selection (IDSS) [Shang 21]

Example: $k = 3$, $|\mathcal{X}| = 4$, **and** $|$**archive**$| = 7$



$\mathcal{X} = \{\bullet^3, \bullet^4, ■^5, \bullet^6\}$, $|\mathcal{X}| > 3$

$\bullet^4$ is the most crowded

$\bullet^4$ is removed from $\mathcal{X}$

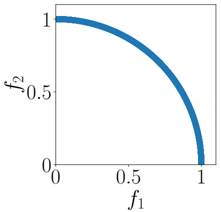
$\mathcal{X} = \{\bullet^3, ■^5, \bullet^6\}$, $|\mathcal{X}| = 3$

Ke Shang, Hisao Ishibuchi, Yang Nan: Distance-based subset selection revisited. GECCO 2021: 439-447

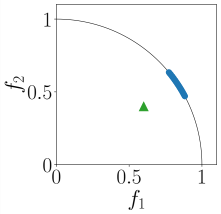
Experimental setup: We used...

- The ND-Tree-based method for nondom. sorting [Jaszkiewicz 18]
- Six representative PBEMO algorithms
  - R-NSGA-II, r-NSGA-II, g-NSGA-II, PBEA, R-MEAD2, MOEA/D-NUMS
- The maximum number of function evaluations = $50,000$
- DTLZ1–4 with 2–6 objectives ☹, the radius of the ROI $r = 0.1$
- $k = 100$, a single reference point near the center of the PF
- An IGD⁺ version of IGD-C [Mohammadi 14] as a quality indicator

Ref. set of IGD



Ref. set of IGD-C



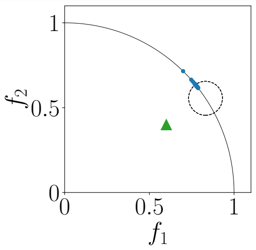
Andrzej Jaszkiewicz, Thibaut Lust: ND-Tree-Based Update: A Fast Algorithm for the Dynamic Nondominance Problem. IEEE Trans. Evol. Comput. 22(5): 778-791 (2018)

Asad Mohammadi, Mohammad Nabi Omidvar, Xiaodong Li, Kalyanmoy Deb: Integrating user preferences and decomposition methods for many-objective optimization. IEEE Congress on Evolutionary Computation 2014: 421-428

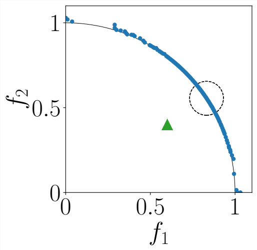
Distributions of points found by R-NSGA-II with $\mu = 100$ (bi-obj. DTLZ2)

The unb. archive pp by the proposed method performs the best

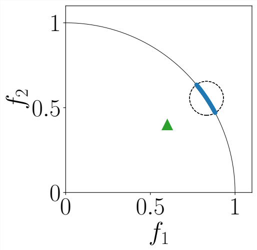
Final population



Arch. pp by IDSS



Arch. pp by Proposed



Interesting findings

- A PBEMO algorithm generates diverse solutions outside the ROI
- Many irrelevant solutions are included in the unbounded archive
- An existing method (IDSS) cannot remove the irrelevant solutions
- It is important for pp methods to consider the DM's pref. info.

Average IGD$^+$-C values of the three solution subsets found by R-NSGA-II

The unb. archive pp by the proposed method performs the best for DTLZ1–4 with $m \in \{2, 3, 4, 5, 6\}$ in most cases

Problem	m	Population	Arch./IDSS	Arch./Proposed
DTLZ1	2	0.0236	0.0018 (+)	0.0012 (+, +)
	3	0.0334	0.0211 (+)	0.0220 (+, ≈)
	4	0.0562	0.0743 (−)	0.0442 (+, +)
	5	0.0933	0.0782 (+)	0.0558 (+, +)
	6	0.1131	0.0779 (+)	0.0695 (+, +)
DTLZ2	2	0.0411	0.0016 (+)	0.0004 (+, +)
	3	0.1247	0.0276 (+)	0.0114 (+, +)
	4	0.1986	0.0720 (+)	0.0339 (+, +)
	5	0.2729	0.1162 (+)	0.0600 (+, +)
	6	0.2840	0.1540 (+)	0.0853 (+, +)
DTLZ3	2	0.0345	0.0083 (+)	0.0078 (+, ≈)
	3	0.1083	0.0460 (+)	0.0309 (+, +)
	4	0.1988	0.1736 (≈)	0.0636 (+, +)
	5	0.2370	0.2169 (≈)	0.1024 (+, +)
	6	0.8749	0.8287 (+)	0.7224 (+, +)
DTLZ4	2	0.1014	0.0829 (≈)	0.0818 (≈, ≈)
	3	0.0838	0.0528 (+)	0.0375 (+, +)
	4	0.1030	0.0741 (+)	0.0465 (+, +)
	5	0.3757	0.1206 (+)	0.0759 (+, +)
	6	0.3214	0.1144 (+)	0.0655 (+, +)

The best population size in R-NSGA-II on all DTLZ1–DTLZ4

- The pop. size $\in \{$ 8, 20, 40, 100, 200, 300, 400, 500 $\}$
- The unbounded archive pp by the proposed method is used

A small pop. size is effective even for many obj. and a large budget

Obj.	1 000 FEs	5 000 FEs	10 000 FEs	30 000 FEs	50 000 FEs
2-obj	8	20	8	8	200
4-obj	8	40	20	20	20
6-obj	20	40	100	40	100

Most previous studies may have used a too large population size

PBEMO	Population size	Obj.	Budget of FEs
R-NSGA-II	100 – 500	2 – 10	$5 \times 10^4 - 2.5 \times 10^5$
PBEA	20, 200	2, 5	2×10^3, 2×10^4
R-MEAD2	200 – 350	4 – 10	$6 \times 10^4, 9 \times 10^4, 1.05 \times 10^5$
NUMS	100 – 660	2 – 10	$4 \times 10^4 - 1.188 \times 10^6$

Conclusion

1. **This work proposed a preference-based postprocessing method**
   - The method can handle the DM's preference information
2. **It investigated effects of the unbounded archive in PBEMO**
   - The arch. and the proposed pp method can improve PBEMO algs
3. **It investigated the best population size for PBEMO**
   - A smaller population size than commonly used is effective even for
     - a large budget of function evaluations (in some cases)
     - many objectives (in some cases)

Future work

- analyzes PBEMO algorithms using other pref. elicitation methods
- extends the proposed pp method for interactive PBEMO

Why is a small population size is effective for PBEMO in this work?

1. **This work used the unb. archive and the preference-based postprocessing method**
   - A small-size population cannot maintain many solutions
   - This issue can be addressed by using the unb. archive and the ppp method
2. **This work used a single reference point**
   - When using multiple reference points, the best pop. size may be large
   - But, I believe that the number of reference points is not problematic
   - Suppose that two reference points are used
   - I expect that the best pop. size for two reference points is just two times larger than the best pop. size for a single reference point

The best pop. size for two reference points may be just two times larger than the best pop. size for a single reference point

The best pop. size for a single reference point

Obj.	$1\,000$ FEs	$5\,000$ FEs	$10\,000$ FEs	$30\,000$ FEs	$50\,000$ FEs
2-obj	8	20	8	8	200
4-obj	8	40	20	20	20
6-obj	20	40	100	40	100

The **EXPECTED** best pop. size for two reference points

Obj.	$1\,000$ FEs	$5\,000$ FEs	$10\,000$ FEs	$30\,000$ FEs	$50\,000$ FEs
2-obj	16	40	16	16	400
4-obj	16	80	40	40	40
6-obj	40	80	200	80	200