A Supplementary File for "A Framework to Handle Multi-modal Multi-objective Optimization in Decomposition-based Evolutionary Algorithms"

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Algorithm S.1: MOEA/D-AGR

note : Although the original MOEA/D-AGR uses the DE variation operator, this version uses the SBX operator.

parameter : $K^{\max}$ is the maximum value of K, and γ controls the schedule of K. The default values of $K^{\max}$ and γ are as follows: $K^{\max} = 0.4N$ and $\gamma = 0.25$.

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the weight vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
    Set the neighborhood index list $\boldsymbol{B}_j = \{b_{j,1}, ..., b_{j,S}\}$;
while The termination criteria are not met **do**
    for $i \in \{1, ..., \mu\}$ **do**
        if $\mathrm{rand}[0, 1] \leq \delta$ **then**
            $\boldsymbol{T} \leftarrow \boldsymbol{B}_i$;
        else
            $\boldsymbol{T} \leftarrow \{1, ..., \mu\}$;
        Randomly select the parent indices a and b from $\boldsymbol{T}$;
        Generate the child $\boldsymbol{u}$ by recombining $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
        Apply the mutation operator to $\boldsymbol{u}$;
        Normalize all objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu)$, and $\boldsymbol{f}(\boldsymbol{u})$;
        $j \leftarrow \underset{k \in \{1,...,N\}}{\mathrm{argmin}} \left\{ g^{\mathrm{tch}}(\boldsymbol{u}|\boldsymbol{w}_k) \right\}$;
        $K \leftarrow \left\lceil \frac{K^{\max}}{1+\exp\left(-20\left((t/t^{\max})-\gamma\right)\right)} \right\rceil$;
        for $k \in \{1, ..., K\}$ **do**
            $l \leftarrow b_{j,k}$;
            if $g^{\mathrm{tch}}(\boldsymbol{u}|\boldsymbol{w}_l) \leq g^{\mathrm{tch}}(\boldsymbol{x}_l|\boldsymbol{w}_l)$ **then**
                $\boldsymbol{x}_l \leftarrow \boldsymbol{u}$;
    $t \leftarrow t + 1$;

Algorithm S.2: MOEA/D-DU

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the weight vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
    Set the neighborhood index list $\boldsymbol{B}_j = \{b_{j,1}, ..., b_{j,S}\}$;
while The termination criteria are not met **do**
    for $i \in \{1, ..., \mu\}$ **do**
        if $\mathrm{rand}[0, 1] \leq \delta$ **then**
            $\boldsymbol{T} \leftarrow \boldsymbol{B}_i$;
        else
            $\boldsymbol{T} \leftarrow \{1, ..., \mu\}$;
        Randomly select the parent indices a and b from $\boldsymbol{T}$;
        Generate the child $\boldsymbol{u}$ by recombining $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
        Apply the mutation operator to $\boldsymbol{u}$;
        Normalize all objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu)$, and $\boldsymbol{f}(\boldsymbol{u})$;
        for $i \in \{1, ..., \mu\}$ **do**
            $d_i \leftarrow \mathrm{PD}(\boldsymbol{f}^{'}(\boldsymbol{u}), \boldsymbol{w}_i)$;
        Set K indices with the minimum d values to $\boldsymbol{R}$;
        while $j \in \boldsymbol{R}$ **do**
            if $g^{\mathrm{dtch}}(\boldsymbol{u}|\boldsymbol{w}_j) \leq g^{\mathrm{dtch}}(\boldsymbol{x}_j|\boldsymbol{w}_j)$ **then**
                $\boldsymbol{x}_j \leftarrow \boldsymbol{u}$;
    $t \leftarrow t + 1$;

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Algorithm S.3: eMOEA/D

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the weight vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
  Set the neighborhood index list $\boldsymbol{B}_j = \{b_{j,1}, ..., b_{j,S}\}$;
while The termination criteria are not met **do**
  for $i \in \{1, ..., \mu\}$ **do**
    Randomly select the parent indices a and b from $\boldsymbol{B}_i$;
    Generate the child $\boldsymbol{u}$ by recombining $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
    Apply the mutation operator to $\boldsymbol{u}$;
    Normalize all objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu)$, and $\boldsymbol{f}(\boldsymbol{u})$;
    Update α for g^{msf} using (5) in the main paper;
    for $j \in \{1, ..., \mu\}$ **do**
      Calculate $g^{\mathrm{msf}}(\boldsymbol{u}|\boldsymbol{w}_j)$;
    Set K indices with the minimum g^{msf} values to $\boldsymbol{R}$;
    while $j \in \boldsymbol{R}$ **do**
      if $g^{\mathrm{msf}}(\boldsymbol{u}|\boldsymbol{w}_j) \leq g^{\mathrm{msf}}(\boldsymbol{x}_j|\boldsymbol{w}_j)$ **then**
        $\boldsymbol{x}_j \leftarrow \boldsymbol{u}$;
  $t \leftarrow t + 1$;

Algorithm S.4: NSGA-III

parameter: $\boldsymbol{F}_l$ is the last front. ρ_j is the number of individuals in the population $\boldsymbol{P}$ that have been assigned to the j-th subproblem ($j \in \{1, ..., N\}$). τ_j is the number of individuals in the last front $\boldsymbol{F}_l$ that have been assigned to the j-th subproblem. $a_{\boldsymbol{x}}$ is the subproblem index which the individual $\boldsymbol{x}$ has been assigned. b_j is a Boolean value. If $b_j = \mathrm{TRUE}$, no individual in the last front $\boldsymbol{F}_l$ has been assigned to the j-th subproblem (i.e., $\tau_j = 0$). If $b_j = \mathrm{FALSE}$, at least one individual in the last front $\boldsymbol{F}_l$ has been assigned to the j-th subproblem (i.e., $\tau_j > 0$).

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the reference vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
while The termination criteria are not met **do**
  A set of children $\boldsymbol{Q} \leftarrow \emptyset$;
  for $j \in \{1, ..., \mu\}$ **do**
    Randomly select the parent indices a and b from $\{1, ..., \mu\}$;
    Generate the child $\boldsymbol{u}$ by recombining $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
    Apply the mutation operator to $\boldsymbol{u}$;
    $\boldsymbol{Q} \leftarrow \boldsymbol{Q} \cup \{\boldsymbol{u}\}$;
  $\boldsymbol{F}_1, \boldsymbol{F}_2, ... \leftarrow \mathrm{nondominatedSorting}(\boldsymbol{P} \cup \boldsymbol{Q})$;
  $i \leftarrow 1$, $\boldsymbol{P} \leftarrow \emptyset$;
  while $|\boldsymbol{P}| < \mu$ **do**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \boldsymbol{F}_i$, $i \leftarrow i + 1$;
  $l \leftarrow i$;
  if $|\boldsymbol{P}| > \mu$ **then**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \setminus \boldsymbol{F}_l$;
    Normalize the objective vector of each $\boldsymbol{x} \in \boldsymbol{P} \cup \boldsymbol{F}_l$;
    for $j \in \{1, ..., \mu\}$ **do**
      $\rho_j \leftarrow 0$, $\tau_j \leftarrow 0$, $b_j \leftarrow \mathrm{FALSE}$;
    for $\boldsymbol{x} \in \boldsymbol{P}$ **do**
      $j \leftarrow \underset{k \in \{1,...,\mu\}}{\mathrm{argmin}} \{\mathrm{PD}(\boldsymbol{f}'(\boldsymbol{x}), \boldsymbol{w}_k)\}$;
      $\rho_j \leftarrow \rho_j + 1$, $a_{\boldsymbol{x}} \leftarrow j$;
    for $\boldsymbol{x} \in \boldsymbol{F}_l$ **do**
      $j \leftarrow \underset{k \in \{1,...,\mu\}}{\mathrm{argmin}} \{\mathrm{PD}(\boldsymbol{f}'(\boldsymbol{x}), \boldsymbol{w}_k)\}$;
      $\tau_j \leftarrow \tau_j + 1$, $a_{\boldsymbol{x}} \leftarrow j$;
    while $|\boldsymbol{P}| < \mu$ **do**
      $j \leftarrow \underset{k \in \{1,...,\mu\} | b_k = \mathrm{FALSE}}{\mathrm{argmin}} \{\rho_k\}$;
      if $\tau_j = 0$ **then**
        $b_j \leftarrow \mathrm{TRUE}$;
      else if $\tau_j > 0$ and $\rho_j = 0$ **then**
        $\boldsymbol{x} \leftarrow \underset{\boldsymbol{y} \in \boldsymbol{F}_l | a_{\boldsymbol{y}} = j}{\mathrm{argmin}} \{\mathrm{PD}(\boldsymbol{f}'(\boldsymbol{y}), \boldsymbol{w}_j)\}$;
        $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{x}\}$, $\boldsymbol{F}_l \leftarrow \boldsymbol{F}_l \setminus \{\boldsymbol{x}\}$, $\rho_j \leftarrow \rho_j + 1$, $\tau_j \leftarrow \tau_j - 1$;
      else
        $\boldsymbol{x} \leftarrow$ Randomly select from $\{\boldsymbol{y} \in \boldsymbol{F}_l | a_{\boldsymbol{y}} = j\}$;
        $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{x}\}$, $\boldsymbol{F}_l \leftarrow \boldsymbol{F}_l \setminus \{\boldsymbol{x}\}$, $\rho_j \leftarrow \rho_j + 1$, $\tau_j \leftarrow \tau_j - 1$;
  $t \leftarrow t + 1$;

Algorithm S.5: θ-DEA

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the reference vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;

while The termination criteria are not met **do**

  A set of children $\boldsymbol{Q} \leftarrow \emptyset$;

  for $j \in \{1, ..., \mu\}$ **do**

    Randomly select the parent indices a and b from $\{1, ..., \mu\}$;

    Generate the child $\boldsymbol{u}$ by recombining $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;

    Apply the mutation operator to $\boldsymbol{u}$;

    $\boldsymbol{Q} \leftarrow \boldsymbol{Q} \cup \{\boldsymbol{u}\}$;

  $\boldsymbol{F}_1, \boldsymbol{F}_2, ... \leftarrow \mathrm{nondominatedSorting}(\boldsymbol{P} \cup \boldsymbol{Q})$;

  $i \leftarrow 1$, $\boldsymbol{S} \leftarrow \emptyset$;

  while $|\boldsymbol{S}| < \mu$ **do**

    $\boldsymbol{S} \leftarrow \boldsymbol{S} \cup \boldsymbol{F}_i$, $i \leftarrow i + 1$;

  $\boldsymbol{F}_1, \boldsymbol{F}_2, ... \leftarrow \theta\text{-}\mathrm{NondominatedSorting}(\boldsymbol{S})$;

  $i \leftarrow 1$, $\boldsymbol{P} \leftarrow \emptyset$;

  while $|\boldsymbol{P}| + |\boldsymbol{F}_i| \leq \mu$ **do**

    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \boldsymbol{F}_i$, $i \leftarrow i + 1$;

  while $|\boldsymbol{P}| < \mu$ **do**

    Randomly select an individual $\boldsymbol{x}$ from $\boldsymbol{F}_i$;

    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{x}\}$, $\boldsymbol{F}_i \leftarrow \boldsymbol{F}_i \setminus \{\boldsymbol{x}\}$;

  $t \leftarrow t + 1$;

Algorithm S.6: RVEA

parameter: $\boldsymbol{v}_j^{\mathrm{init}}$ is the initial unit reference vector of the j-th subproblem ($j \in \{1, ..., \mu\}$). $\boldsymbol{v}_j$ is the unit reference vector of the j-th subproblem. $a_{\boldsymbol{x}}$ is the subproblem index which the individual $\boldsymbol{x}$ has been assigned. The "mod" operator is the modulo operator. t^{freq} controls the frequency of adjusting the unit reference vectors. The default t^{freq} value is $0.1t^{\mathrm{max}}$. The "$\circ$" operator is the Hadamard product.

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the reference vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;

for $j \in \{1, ..., N\}$ **do**

  $\boldsymbol{v}_j^{\mathrm{init}} \leftarrow \frac{\boldsymbol{w}_j}{\|\boldsymbol{w}_j\|}$, $\boldsymbol{v}_j \leftarrow \boldsymbol{v}_j^{\mathrm{init}}$;

while The termination criteria are not met **do**

  $\mu \leftarrow |\boldsymbol{P}|$;

  A set of children $\boldsymbol{Q} \leftarrow \emptyset$;

  $\boldsymbol{T} = \{1, ..., \mu\}$;

  for $i \in \{1, ..., \mu/2\}$ **do**

    Randomly select the parent indices a and b from $\boldsymbol{T}$;

    $\boldsymbol{T} \leftarrow \boldsymbol{T} \setminus \{a, b\}$;

    Generate the children $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ by recombining $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;

    Apply the mutation operator to $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$;

    $\boldsymbol{Q} \leftarrow \boldsymbol{Q} \cup \{\boldsymbol{u}_1, \boldsymbol{u}_2\}$;

  $\boldsymbol{R} \leftarrow \boldsymbol{P} \cup \boldsymbol{Q}$;

  for $\boldsymbol{x} \in \boldsymbol{R}$ **do**

    $\boldsymbol{f}'(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{z}^*$;

    $j \leftarrow \underset{k \in \{1, ..., N\}}{\mathrm{argmin}} \left\{ \mathrm{angle}\big(\boldsymbol{f}'(\boldsymbol{x}), \boldsymbol{v}_k\big) \right\}$;

    $a_{\boldsymbol{x}} \leftarrow j$;

  $\boldsymbol{P} \leftarrow \emptyset$;

  for $j \in \{1, ..., N\}$ **do**

    $\boldsymbol{x} \leftarrow \underset{\boldsymbol{y} \in \boldsymbol{R}|a_{\boldsymbol{y}} = j}{\mathrm{argmin}} \{\mathrm{APD}(\boldsymbol{y}, \boldsymbol{v}_j)\}$;

    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{x}\}$;

  if $t \bmod t^{\mathrm{freq}} = 0$ **then**

    Set $\boldsymbol{z}^{\mathrm{min}}$ and $\boldsymbol{z}^{\mathrm{max}}$ to the minimal and maximum objective values in $\boldsymbol{P}$;

    for $j \in \{1, ..., N\}$ **do**

      $\boldsymbol{v}_j = \frac{\boldsymbol{v}_j^{\mathrm{init}} \circ (\boldsymbol{z}^{\mathrm{max}} - \boldsymbol{z}^{\mathrm{min}})}{\|\boldsymbol{v}_j^{\mathrm{init}} \circ (\boldsymbol{z}^{\mathrm{max}} - \boldsymbol{z}^{\mathrm{min}})\|}$;

  $t \leftarrow t + 1$;

Algorithm S.7: MOEA/D-AGR-ADA

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the weight vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ g^{\mathrm{tch}}(\boldsymbol{x}_i | \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{x}_i$ to the j-th subproblem;
while The termination criteria are not met **do**
  $\mu \leftarrow |\boldsymbol{P}|$;
  Randomly select a and b from $\{1, ..., \mu\}$ such that $a \neq b$;
  Generate the child $\boldsymbol{u}$ by applying the crossover operation to $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
  Apply the mutation operation to $\boldsymbol{u}$;
  Normalize the objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu), \boldsymbol{f}(\boldsymbol{u})$;
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ g^{\mathrm{tch}}(\boldsymbol{u} | \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{u}$ to the j-th subproblem;
  $\boldsymbol{X} \leftarrow \{\boldsymbol{x} \in \boldsymbol{P} | \boldsymbol{x}$ has been assigned to the j-th subproblem and is in the neighborhood of $\boldsymbol{u}$ in the solution space$\}$;
  $b^{\mathrm{explorer}} \leftarrow \mathrm{FALSE}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{FALSE}$;
  if $\boldsymbol{X} = \emptyset$ **then**
    $b^{\mathrm{explorer}} \leftarrow \mathrm{TRUE}$;
  for $\boldsymbol{x} \in \boldsymbol{X}$ **do**
    if $g^{\mathrm{tch}}(\boldsymbol{u} | \boldsymbol{w}_j) \leq g^{\mathrm{tch}}(\boldsymbol{x} | \boldsymbol{w}_j)$ **then**
      $\boldsymbol{P} \leftarrow \boldsymbol{P} \setminus \{\boldsymbol{x}\}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{TRUE}$
  if $b^{\mathrm{winner}} = \mathrm{TRUE}$ or $b^{\mathrm{explorer}} = \mathrm{TRUE}$ **then**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{u}\}$;
  $t \leftarrow t + 1$;
return $\boldsymbol{P}$ for the decision making (Subsection III-G) or benchmarking (Subsection III-H);

Algorithm S.8: MOEA/D-DU-ADA

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the weight vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ \mathrm{PD}(\boldsymbol{f}'(\boldsymbol{x}_i), \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{x}_i$ to the j-th subproblem;
while The termination criteria are not met **do**
  $\mu \leftarrow |\boldsymbol{P}|$;
  Randomly select a and b from $\{1, ..., \mu\}$ such that $a \neq b$;
  Generate the child $\boldsymbol{u}$ by applying the crossover operation to $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
  Apply the mutation operation to $\boldsymbol{u}$;
  Normalize the objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu), \boldsymbol{f}(\boldsymbol{u})$;
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ \mathrm{PD}(\boldsymbol{f}'(\boldsymbol{u}), \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{u}$ to the j-th subproblem;
  $\boldsymbol{X} \leftarrow \{\boldsymbol{x} \in \boldsymbol{P} | \boldsymbol{x}$ has been assigned to the j-th subproblem and is in the neighborhood of $\boldsymbol{u}$ in the solution space$\}$;
  $b^{\mathrm{explorer}} \leftarrow \mathrm{FALSE}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{FALSE}$;
  if $\boldsymbol{X} = \emptyset$ **then**
    $b^{\mathrm{explorer}} \leftarrow \mathrm{TRUE}$;
  for $\boldsymbol{x} \in \boldsymbol{X}$ **do**
    if $g^{\mathrm{dtch}}(\boldsymbol{u} | \boldsymbol{w}_j) \leq g^{\mathrm{dtch}}(\boldsymbol{x} | \boldsymbol{w}_j)$ **then**
      $\boldsymbol{P} \leftarrow \boldsymbol{P} \setminus \{\boldsymbol{x}\}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{TRUE}$
  if $b^{\mathrm{winner}} = \mathrm{TRUE}$ or $b^{\mathrm{explorer}} = \mathrm{TRUE}$ **then**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{u}\}$;
  $t \leftarrow t + 1$;
return $\boldsymbol{P}$ for the decision making (Subsection III-G) or benchmarking (Subsection III-H);

Algorithm S.9: eMOEA/D-ADA

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the weight vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ g^{\mathrm{msf}}(\boldsymbol{x}_i | \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{x}_i$ to the j-th subproblem;
while The termination criteria are not met **do**
  $\mu \leftarrow |\boldsymbol{P}|$;
  Randomly select a and b from $\{1, ..., \mu\}$ such that $a \neq b$;
  Generate the child $\boldsymbol{u}$ by applying the crossover operation to $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
  Apply the mutation operation to $\boldsymbol{u}$;
  Normalize the objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu), \boldsymbol{f}(\boldsymbol{u})$;
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ g^{\mathrm{msf}}(\boldsymbol{u} | \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{u}$ to the j-th subproblem;
  $\boldsymbol{X} \leftarrow \{\boldsymbol{x} \in \boldsymbol{P} | \boldsymbol{x}$ has been assigned to the j-th subproblem and is in the neighborhood of $\boldsymbol{u}$ in the solution space$\}$;
  $b^{\mathrm{explorer}} \leftarrow \mathrm{FALSE}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{FALSE}$;
  if $\boldsymbol{X} = \emptyset$ **then**
    $b^{\mathrm{explorer}} \leftarrow \mathrm{TRUE}$;
  for $\boldsymbol{x} \in \boldsymbol{X}$ **do**
    if $g^{\mathrm{msf}}(\boldsymbol{u} | \boldsymbol{w}_j) \leq g^{\mathrm{msf}}(\boldsymbol{x} | \boldsymbol{w}_j)$ **then**
      $\boldsymbol{P} \leftarrow \boldsymbol{P} \setminus \{\boldsymbol{x}\}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{TRUE}$
  if $b^{\mathrm{winner}} = \mathrm{TRUE}$ or $b^{\mathrm{explorer}} = \mathrm{TRUE}$ **then**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{u}\}$;
  $t \leftarrow t + 1$;
return $\boldsymbol{P}$ for the decision making (Subsection III-G) or benchmarking (Subsection III-H);

Algorithm S.10: NSGA-III-ADA

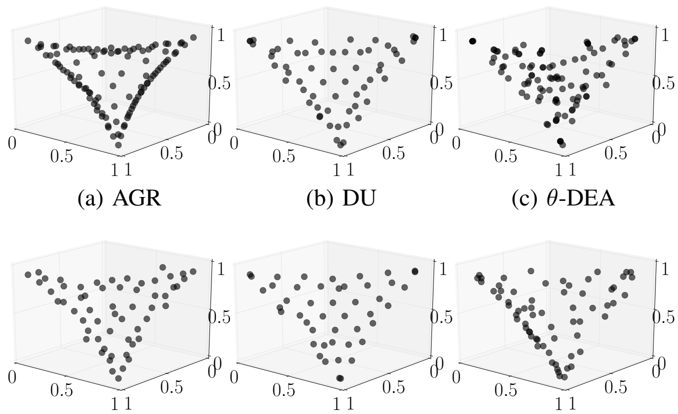
$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the reference vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ \mathrm{PD}(\boldsymbol{f}'(\boldsymbol{x}_i), \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{x}_i$ to the j-th subproblem;
while The termination criteria are not met **do**
  $\mu \leftarrow |\boldsymbol{P}|$;
  Randomly select a and b from $\{1, ..., \mu\}$ such that $a \neq b$;
  Generate the child $\boldsymbol{u}$ by applying the crossover operation to $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
  Apply the mutation operation to $\boldsymbol{u}$;
  Normalize the objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu), \boldsymbol{f}(\boldsymbol{u})$;
  $j \leftarrow \underset{k \in \{1,...,N\}}{\operatorname{argmin}} \left\{ \mathrm{PD}(\boldsymbol{f}'(\boldsymbol{u}), \boldsymbol{w}_k) \right\}$;
  Assign $\boldsymbol{u}$ to the j-th subproblem;
  $\boldsymbol{X} \leftarrow \{\boldsymbol{x} \in \boldsymbol{P} | \boldsymbol{x}$ has been assigned to the j-th subproblem and is in the neighborhood of $\boldsymbol{u}$ in the solution space$\}$;
  $b^{\mathrm{explorer}} \leftarrow \mathrm{FALSE}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{FALSE}$;
  if $\boldsymbol{X} = \emptyset$ **then**
    $b^{\mathrm{explorer}} \leftarrow \mathrm{TRUE}$;
  for $\boldsymbol{x} \in \boldsymbol{X}$ **do**
    if ($\boldsymbol{u}$ dominates $\boldsymbol{x}$) or ($\boldsymbol{u}$ and $\boldsymbol{x}$ are non-dominated, and $\mathrm{PD}(\boldsymbol{f}'(\boldsymbol{u}), \boldsymbol{w}_j) < \mathrm{PD}(\boldsymbol{f}'(\boldsymbol{x}), \boldsymbol{w}_j)$) **then**
      $\boldsymbol{P} \leftarrow \boldsymbol{P} \setminus \{\boldsymbol{x}\}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{TRUE}$
  if $b^{\mathrm{winner}} = \mathrm{TRUE}$ or $b^{\mathrm{explorer}} = \mathrm{TRUE}$ **then**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{u}\}$;
  $t \leftarrow t + 1$;
return $\boldsymbol{P}$ for the decision making (Subsection III-G) or benchmarking (Subsection III-H);

Algorithm S.11: θ-DEA-ADA

$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the reference vector set $\boldsymbol{W} = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_N\}$;
for $i \in \{1, ..., \mu\}$ **do**
  $j \leftarrow \underset{k \in \{1,...,N\}}{\mathrm{argmin}} \left\{\mathrm{PD}(\boldsymbol{f}'(\boldsymbol{x}_i), \boldsymbol{w}_k)\right\}$;
  Assign $\boldsymbol{x}_i$ to the j-th subproblem;
while The termination criteria are not met **do**
  $\mu \leftarrow |\boldsymbol{P}|$;
  Randomly select a and b from $\{1, ..., \mu\}$ such that $a \neq b$;
  Generate the child $\boldsymbol{u}$ by applying the crossover operation to $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
  Apply the mutation operation to $\boldsymbol{u}$;
  Normalize the objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu), \boldsymbol{f}(\boldsymbol{u})$;
  $j \leftarrow \underset{k \in \{1,...,N\}}{\mathrm{argmin}} \left\{\mathrm{PD}(\boldsymbol{f}'(\boldsymbol{u}), \boldsymbol{w}_k)\right\}$;
  Assign $\boldsymbol{u}$ to the j-th subproblem;
  $\boldsymbol{X} \leftarrow \{\boldsymbol{x} \in \boldsymbol{P} | \boldsymbol{x}$ has been assigned to the j-th subproblem and is in the neighborhood of $\boldsymbol{u}$ in the solution space$\}$;
  $b^{\mathrm{explorer}} \leftarrow \mathrm{FALSE}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{FALSE}$;
  if $\boldsymbol{X} = \emptyset$ **then**
    $b^{\mathrm{explorer}} \leftarrow \mathrm{TRUE}$;
  for $\boldsymbol{x} \in \boldsymbol{X}$ **do**
    if ($\boldsymbol{u}$ dominates $\boldsymbol{x}$) or ($\boldsymbol{u}$ and $\boldsymbol{x}$ are non-dominated, and $\boldsymbol{u}$ θ-dominates $\boldsymbol{x}$) **then**
      $\boldsymbol{P} \leftarrow \boldsymbol{P} \setminus \{\boldsymbol{x}\}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{TRUE}$
  if $b^{\mathrm{winner}} = \mathrm{TRUE}$ or $b^{\mathrm{explorer}} = \mathrm{TRUE}$ **then**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{u}\}$;
  $t \leftarrow t + 1$;
return $\boldsymbol{P}$ for the decision making (Subsection III-G) or benchmarking (Subsection III-H);

Algorithm S.12: RVEA-ADA

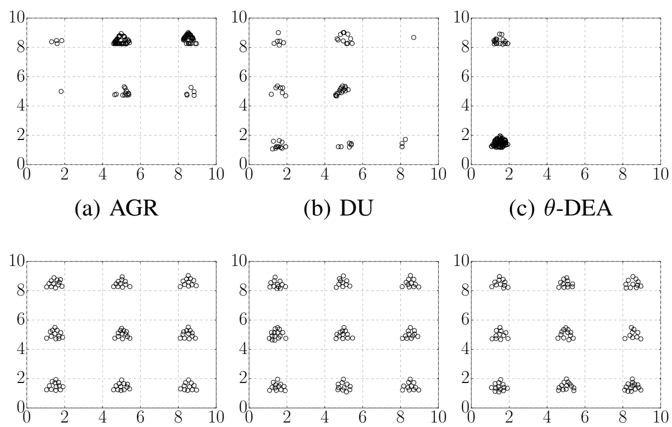
$t \leftarrow 1$, $\mu \leftarrow N$, initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, ..., \boldsymbol{x}_\mu\}$ and the reference vector set $\boldsymbol{V} = \{\boldsymbol{v}_1, ..., \boldsymbol{v}_N\}$;
/* This initial assignment operation is actually unnecessary in RVEA-ADA */
for $i \in \{1, ..., \mu\}$ **do**
  $j \leftarrow \underset{k \in \{1,...,N\}}{\mathrm{argmin}} \left\{\mathrm{angle}(\boldsymbol{f}'(\boldsymbol{x}_i), \boldsymbol{v}_k)\right\}$;
  Assign $\boldsymbol{x}_i$ to the j-th subproblem;
while The termination criteria are not met **do**
  $\mu \leftarrow |\boldsymbol{P}|$;
  Randomly select a and b from $\{1, ..., \mu\}$ such that $a \neq b$;
  Generate the child $\boldsymbol{u}$ by applying the crossover operation to $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$;
  Apply the mutation operation to $\boldsymbol{u}$;
  Normalize the objective vectors $\boldsymbol{f}(\boldsymbol{x}_1), ..., \boldsymbol{f}(\boldsymbol{x}_\mu), \boldsymbol{f}(\boldsymbol{u})$;
  $j \leftarrow \underset{k \in \{1,...,N\}}{\mathrm{argmin}} \left\{\mathrm{angle}(\boldsymbol{f}'(\boldsymbol{u}), \boldsymbol{v}_k)\right\}$;
  Assign $\boldsymbol{u}$ to the j-th subproblem;
  $\boldsymbol{X} \leftarrow \{\boldsymbol{x} \in \boldsymbol{P} | \boldsymbol{x}$ has been assigned to the j-th subproblem and is in the neighborhood of $\boldsymbol{u}$ in the solution space$\}$;
  $b^{\mathrm{explorer}} \leftarrow \mathrm{FALSE}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{FALSE}$;
  if $\boldsymbol{X} = \emptyset$ **then**
    $b^{\mathrm{explorer}} \leftarrow \mathrm{TRUE}$;
  for $\boldsymbol{x} \in \boldsymbol{X}$ **do**
    if $\mathrm{APD}(\boldsymbol{u}|\boldsymbol{v}_j) \leq \mathrm{APD}(\boldsymbol{x}|\boldsymbol{v}_j)$ **then**
      $\boldsymbol{P} \leftarrow \boldsymbol{P} \setminus \{\boldsymbol{x}\}$ and $b^{\mathrm{winner}} \leftarrow \mathrm{TRUE}$
  if $b^{\mathrm{winner}} = \mathrm{TRUE}$ or $b^{\mathrm{explorer}} = \mathrm{TRUE}$ **then**
    $\boldsymbol{P} \leftarrow \boldsymbol{P} \cup \{\boldsymbol{u}\}$;
  $t \leftarrow t + 1$;
return $\boldsymbol{P}$ for the decision making (Subsection III-G) or benchmarking (Subsection III-H);



(a) AGR (b) DU (c) θ-DEA

(d) AGR-ADA (e) DU-ADA (f) θ-DEA-ADA

Fig. S.1: Distribution of non-dominated solutions found by each method in the objective space on the three-objective Polygon problem. AGR and DU stand for MOEA/D-AGR and MOEA/D-DU, respectively. The x, y, and z axes represent f_1, f_2, and f_3, respectively.



(a) AGR (b) DU (c) θ-DEA

(d) AGR-ADA (e) DU-ADA (f) θ-DEA-ADA

Fig. S.2: Distribution of non-dominated solutions found by each method in the solution space on the three-objective Polygon problem. AGR and DU stand for MOEA/D-AGR and MOEA/D-DU, respectively. The x and y axes represent x_1 and x_2, respectively.

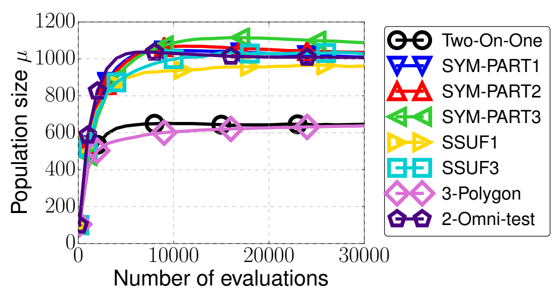
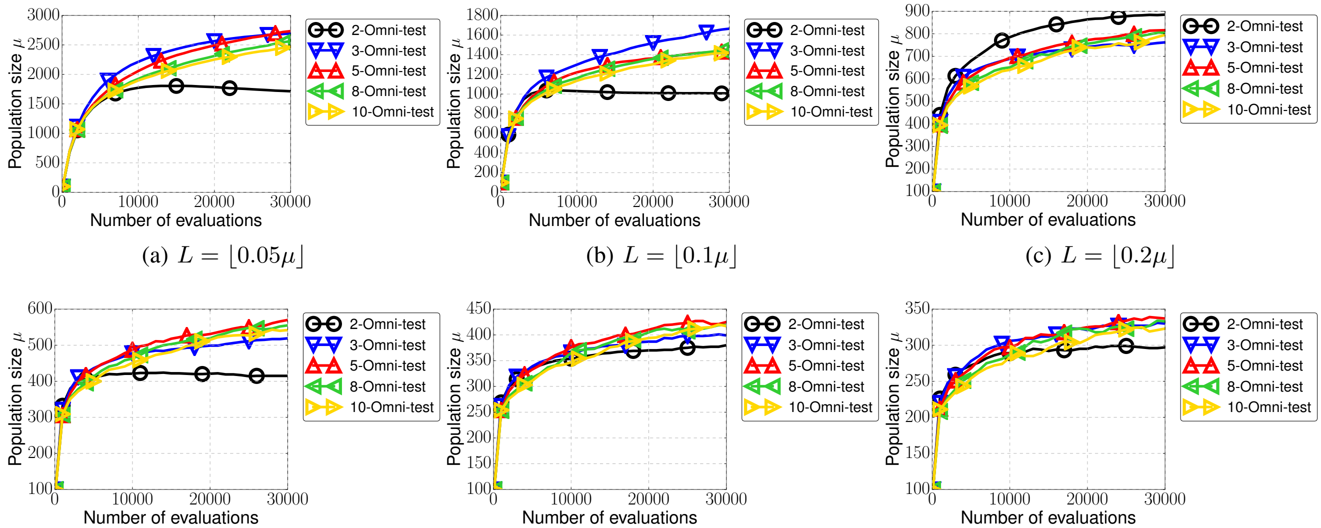


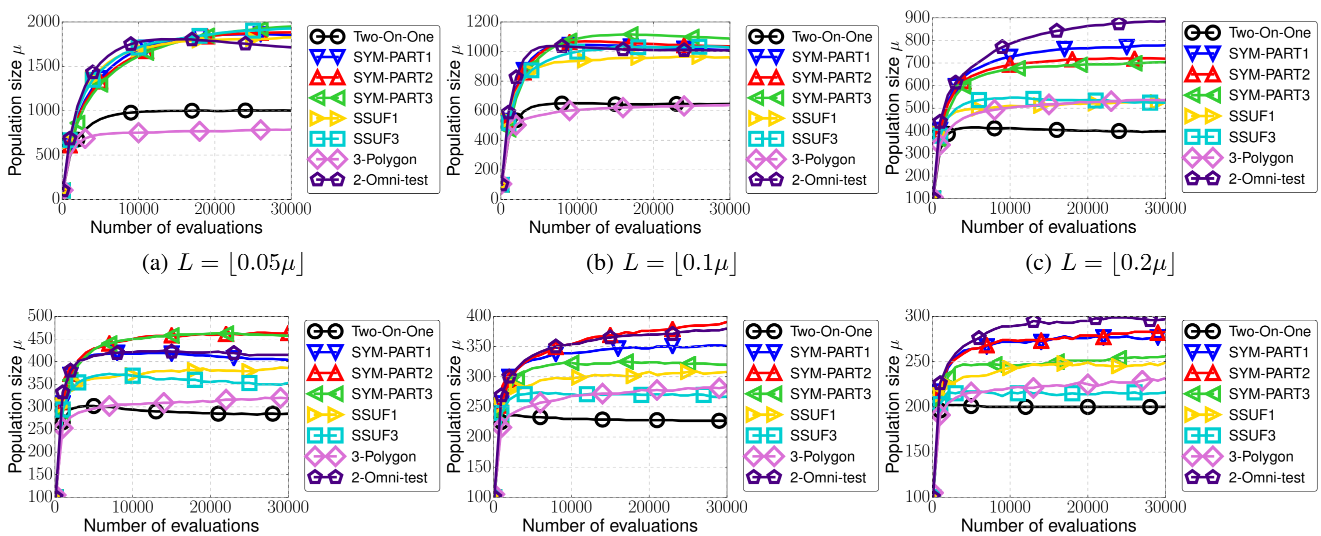
Fig. S.3: Evolution of μ of NSGA-III-ADA on the eight test problems. The mean μ values across 31 runs are reported for each problem.



(a) $L = \lfloor 0.05\mu \rfloor$ (b) $L = \lfloor 0.1\mu \rfloor$ (c) $L = \lfloor 0.2\mu \rfloor$

(d) $L = \lfloor 0.3\mu \rfloor$ (e) $L = \lfloor 0.4\mu \rfloor$ (f) $L = \lfloor 0.5\mu \rfloor$

Fig. S.4: Evolution of μ of NSGA-III-ADA with $L \in \{\lfloor 0.05\mu \rfloor, \lfloor 0.2\mu \rfloor, \lfloor 0.3\mu \rfloor, \lfloor 0.4\mu \rfloor, \lfloor 0.5\mu \rfloor\}$ on Omni-test with $D \in \{2, 3, 5, 8, 10\}$. The mean μ values across 31 runs are reported for each problem.



(a) $L = \lfloor 0.05\mu \rfloor$ (b) $L = \lfloor 0.1\mu \rfloor$ (c) $L = \lfloor 0.2\mu \rfloor$

(d) $L = \lfloor 0.3\mu \rfloor$ (e) $L = \lfloor 0.4\mu \rfloor$ (f) $L = \lfloor 0.5\mu \rfloor$

Fig. S.5: Evolution of μ of NSGA-III-ADA with $L \in \{\lfloor 0.05\mu \rfloor, \lfloor 0.2\mu \rfloor, \lfloor 0.3\mu \rfloor, \lfloor 0.4\mu \rfloor, \lfloor 0.5\mu \rfloor\}$ on the eight test problems. The mean μ values across 31 runs are reported for each problem.

TABLE S.1: Results of the six EMOAs and their ADA versions on the 15 test problem instances. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. “Orig.” and “ADA” represent an original EMOA and its ADA version, respectively. The best results between them are shaded. The symbols $+$, $-$, and $\approx$ indicate that a given ADA version performs significantly better ($+$), significantly worse ($-$), and not significantly better or worse ($\approx$) compared to its original EMOA according to the Wilcoxon rank-sum test with $p < 0.001$.

(a) IGD$^+$

	MOEA/D-AGR		MOEA/D-DU		eMOEA/D		NSGA-III		θ-DEA		RVEA	
	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA
Two-On-One	0.020	0.051$-$	0.027	0.046$-$	0.030	0.053$-$	0.020	0.021$-$	0.020	0.021$-$	0.035	0.048$-$
SYM-PART1	0.010	0.011$-$	0.010	0.011$-$	0.010	0.011$-$	0.010	0.011$-$	0.013	0.013$-$	0.014	0.017$-$
SYM-PART2	0.011	0.013$-$	0.011	0.011$\approx$	0.013	0.012$+$	0.012	0.012$-$	0.013	0.014$\approx$	0.021	0.019$\approx$
SYM-PART3	0.011	0.013$-$	0.011	0.012$-$	0.011	0.012$-$	0.012	0.012$-$	0.013	0.015$-$	0.020	0.020$\approx$
SSUF1	0.003	0.011$-$	0.003	0.010$-$	0.006	0.010$-$	0.003	0.004$-$	0.003	0.004$-$	0.007	0.006$+$
SSUF3	0.012	0.018$-$	0.004	0.017$-$	0.046	0.018$+$	0.008	0.013$-$	0.007	0.012$-$	0.016	0.014$\approx$
3-Polygon	0.033	0.036$-$	0.038	0.040$-$	0.038	0.040$-$	0.027	0.044$-$	0.029	0.040$-$	0.045	0.040$+$
5-Polygon	0.033	0.053$-$	0.105	0.104$\approx$	0.093	0.113$-$	0.036	0.040$\approx$	0.102	0.067$+$	0.124	0.108$+$
8-Polygon	0.059	0.110$-$	0.109	0.170$-$	0.165	0.171$\approx$	0.050	0.118$-$	0.130	0.128$\approx$	0.224	0.165$+$
10-Polygon	0.061	0.089$-$	0.160	0.162$\approx$	0.192	0.156$+$	0.049	0.126$-$	0.147	0.141$\approx$	0.225	0.170$+$
2-Omni-test	0.006	0.006$-$	0.006	0.006$-$	0.006	0.006$\approx$	0.006	0.006$\approx$	0.007	0.007$\approx$	0.010	0.008$+$
3-Omni-test	0.010	0.017$-$	0.009	0.015$-$	0.011	0.014$-$	0.011	0.011$\approx$	0.011	0.011$\approx$	0.025	0.021$+$
5-Omni-test	0.019	0.045$-$	0.018	0.038$-$	0.028	0.036$-$	0.024	0.038$-$	0.023	0.036$-$	0.070	0.077$\approx$
8-Omni-test	0.042	0.147$-$	0.038	0.141$-$	0.073	0.141$-$	0.060	0.148$-$	0.056	0.141$-$	0.161	0.234$-$
10-Omni-test	0.062	0.270$-$	0.060	0.253$-$	0.128	0.267$-$	0.098	0.267$-$	0.102	0.261$-$	0.242	0.388$-$
APS	0.0	1.0	0.0	0.8	0.2	0.667	0.0	0.8	0.067	0.6	0.467	0.267

(b) IGDX

	MOEA/D-AGR		MOEA/D-DU		eMOEA/D		NSGA-III		θ-DEA		RVEA	
	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA
Two-On-One	0.308	0.041$+$	0.207	0.036$+$	0.374	0.040$+$	0.134	0.038$+$	0.084	0.037$+$	0.240	0.066$+$
SYM-PART1	7.129	0.071$+$	5.043	0.064$+$	9.250	0.065$+$	5.084	0.065$+$	6.237	0.064$+$	6.963	0.085$+$
SYM-PART2	6.794	0.082$+$	5.161	0.074$+$	7.970	0.074$+$	5.019	0.077$+$	4.212	0.073$+$	6.612	0.093$+$
SYM-PART3	6.534	0.068$+$	4.677	0.065$+$	6.677	0.063$+$	5.682	0.063$+$	5.571	0.064$+$	7.546	0.078$+$
SSUF1	0.135	0.105$+$	0.114	0.104$\approx$	0.188	0.105$+$	0.120	0.066$+$	0.108	0.065$+$	0.133	0.077$+$
SSUF3	0.184	0.048$+$	0.060	0.043$\approx$	0.282	0.038$+$	0.084	0.024$+$	0.067	0.024$+$	0.087	0.028$+$
3-Polygon	1.408	0.090$+$	0.540	0.092$+$	0.511	0.086$+$	3.073	0.096$+$	3.677	0.095$+$	4.555	0.090$+$
5-Polygon	0.684	0.077$+$	0.396	0.138$+$	0.457	0.184$+$	1.380	0.079$+$	1.594	0.085$+$	4.322	0.100$+$
8-Polygon	0.794	0.116$+$	0.526	0.223$+$	0.649	0.223$+$	2.443	0.136$+$	3.841	0.158$+$	4.653	0.156$+$
10-Polygon	0.830	0.079$+$	0.390	0.195$+$	0.625	0.197$+$	2.003	0.139$+$	2.810	0.164$+$	3.480	0.139$+$
2-Omni-test	0.620	0.026$+$	0.364	0.024$+$	0.965	0.024$+$	0.407	0.024$+$	0.337	0.023$+$	0.759	0.030$+$
3-Omni-test	1.789	0.115$+$	1.280	0.106$+$	2.075	0.103$+$	1.313	0.085$+$	1.162	0.084$+$	1.204	0.135$+$
5-Omni-test	3.162	1.402$+$	2.925	1.404$+$	3.782	1.404$+$	2.698	1.408$+$	2.690	1.386$+$	2.546	1.678$+$
8-Omni-test	4.877	3.160$+$	4.770	3.151$+$	5.376	3.192$+$	4.323	3.207$+$	4.288	3.194$+$	4.285	3.483$+$
10-Omni-test	6.084	4.175$+$	5.510	4.155$+$	6.200	4.193$+$	5.317	4.210$+$	5.289	4.178$+$	5.327	4.455$+$
APS	1.0	0.0	0.867	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0

TABLE S.2: Results of the nine EMMAs on the 15 test problem instances. AGR, DU, TriMOEA, and MO_Ring stand for MOEA/D-AGR, MOEA/D-DU, TriMOEA-TA&R, and MO_Ring_PSO_SCD, respectively. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	TriMOEA	MO_Ring	Omni-optimizer
Two-On-One	0.051	0.046	0.053	0.021	0.021	0.048	0.029	0.030	0.026
SYM-PART1	0.011	0.011	0.011	0.011	0.013	0.017	0.010	0.016	0.012
SYM-PART2	0.013	0.011	0.012	0.012	0.014	0.019	0.012	0.017	0.018
SYM-PART3	0.013	0.012	0.012	0.012	0.015	0.020	0.020	0.018	0.021
SSUF1	0.011	0.010	0.010	0.004	0.004	0.006	0.003	0.005	0.005
SSUF3	0.018	0.017	0.018	0.013	0.012	0.014	0.028	0.008	0.010
3-Polygon	0.036	0.040	0.040	0.044	0.040	0.040	0.040	0.026	0.026
5-Polygon	0.053	0.104	0.113	0.040	0.067	0.108	0.147	0.034	0.034
8-Polygon	0.110	0.170	0.171	0.118	0.128	0.165	0.154	0.058	0.054
10-Polygon	0.089	0.162	0.156	0.126	0.141	0.170	0.170	0.051	0.049
2-Omni-test	0.006	0.006	0.006	0.006	0.007	0.008	0.006	0.008	0.006
3-Omni-test	0.017	0.015	0.014	0.011	0.011	0.021	0.012	0.022	0.011
5-Omni-test	0.045	0.038	0.036	0.038	0.036	0.077	0.038	0.147	0.022
8-Omni-test	0.147	0.141	0.141	0.148	0.141	0.234	0.130	0.663	0.044
10-Omni-test	0.270	0.253	0.267	0.267	0.261	0.388	0.264	1.256	0.062
APS	3.733 (7)	2.800 (5)	2.867 (6)	1.733 (2)	2.667 (4)	5.933 (9)	2.400 (3)	4.200 (8)	1.600 (1)

(b) IGDX

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	TriMOEA	MO_Ring	Omni-optimizer
Two-On-One	0.041	0.036	0.040	0.038	0.037	0.066	0.567	0.037	0.038
SYM-PART1	0.071	0.064	0.065	0.065	0.064	0.085	2.308	0.148	3.803
SYM-PART2	0.082	0.074	0.074	0.077	0.073	0.093	9.929	0.161	1.086
SYM-PART3	0.068	0.065	0.063	0.063	0.064	0.078	5.856	0.491	1.362
SSUF1	0.105	0.104	0.105	0.066	0.065	0.077	0.108	0.086	0.090
SSUF3	0.048	0.043	0.038	0.024	0.024	0.028	0.114	0.020	0.054
3-Polygon	0.090	0.092	0.086	0.096	0.095	0.090	2.702	0.116	0.198
5-Polygon	0.077	0.138	0.184	0.079	0.085	0.100	2.331	0.100	0.112
8-Polygon	0.116	0.223	0.223	0.136	0.158	0.156	2.125	0.131	0.144
10-Polygon	0.079	0.195	0.197	0.139	0.164	0.139	1.989	0.105	0.115
2-Omni-test	0.026	0.024	0.024	0.024	0.023	0.030	2.956	0.050	0.072
3-Omni-test	0.115	0.106	0.103	0.085	0.084	0.135	3.802	0.378	0.532
5-Omni-test	1.402	1.404	1.404	1.408	1.386	1.678	5.175	2.223	2.034
8-Omni-test	3.160	3.151	3.192	3.207	3.194	3.483	6.730	4.148	3.695
10-Omni-test	4.175	4.155	4.193	4.210	4.178	4.455	7.644	5.175	4.684
APS	2.067 (3)	2.133 (4)	2.267 (5)	0.800 (1)	1.000 (2)	4.067 (6)	7.733 (9)	4.333 (7)	5.067 (8)

TABLE S.3: Results of MOEA/D-AGR-ADA with the six L values on the 15 test problem instances. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.049	0.051	0.048	0.049	0.049	0.048
SYM-PART1	0.013	0.011	0.011	0.021	0.024	0.025
SYM-PART2	0.014	0.013	0.012	0.015	0.021	0.025
SYM-PART3	0.015	0.013	0.012	0.013	0.012	0.013
SSUF1	0.011	0.011	0.011	0.012	0.012	0.013
SSUF3	0.018	0.018	0.020	0.022	0.021	0.022
3-Polygon	0.036	0.036	0.036	0.036	0.036	0.036
5-Polygon	0.053	0.053	0.053	0.053	0.053	0.053
8-Polygon	0.111	0.110	0.109	0.110	0.110	0.110
10-Polygon	0.089	0.089	0.089	0.089	0.089	0.090
2-Omni-test	0.007	0.006	0.009	0.009	0.011	0.012
3-Omni-test	0.015	0.017	0.017	0.020	0.022	0.023
5-Omni-test	0.057	0.045	0.043	0.042	0.043	0.040
8-Omni-test	0.212	0.147	0.119	0.108	0.101	0.102
10-Omni-test	0.400	0.270	0.206	0.185	0.169	0.165
APS	1.600 (6)	0.933 (3)	0.600 (1)	0.667 (2)	1.000 (4)	1.267 (5)

(b) IGDX

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.041	0.041	0.039	0.040	0.041	0.042
SYM-PART1	0.079	0.071	0.070	3.989	4.353	1.578
SYM-PART2	0.091	0.082	0.131	2.681	3.785	4.812
SYM-PART3	0.081	0.068	0.094	0.554	1.559	1.930
SSUF1	0.094	0.105	0.112	0.120	0.130	0.138
SSUF3	0.038	0.048	0.056	0.061	0.064	0.077
3-Polygon	0.090	0.090	0.090	0.172	0.240	0.306
5-Polygon	0.077	0.077	0.077	0.099	0.149	0.133
8-Polygon	0.118	0.116	0.119	0.159	0.220	0.206
10-Polygon	0.079	0.079	0.082	0.109	0.143	0.168
2-Omni-test	0.027	0.026	0.033	0.834	0.867	0.546
3-Omni-test	0.086	0.115	0.733	0.814	0.630	1.004
5-Omni-test	1.378	1.402	1.553	1.778	1.807	1.851
8-Omni-test	3.327	3.160	3.146	3.237	3.266	3.333
10-Omni-test	4.382	4.175	4.129	4.140	4.155	4.250
APS	0.800 (2)	0.133 (1)	0.800 (3)	2.600 (4)	2.867 (5)	3.200 (6)

TABLE S.4: Results of MOEA/D-DU-ADA with the six L values on the 15 test problem instances. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.046	0.046	0.043	0.044	0.045	0.043
SYM-PART1	0.011	0.011	0.011	0.020	0.024	0.025
SYM-PART2	0.012	0.011	0.011	0.013	0.019	0.024
SYM-PART3	0.012	0.012	0.011	0.011	0.011	0.012
SSUF1	0.010	0.010	0.011	0.011	0.012	0.013
SSUF3	0.016	0.017	0.018	0.018	0.019	0.019
3-Polygon	0.039	0.040	0.040	0.040	0.040	0.040
5-Polygon	0.105	0.104	0.104	0.104	0.104	0.104
8-Polygon	0.166	0.170	0.166	0.166	0.164	0.166
10-Polygon	0.161	0.162	0.158	0.158	0.158	0.158
2-Omni-test	0.006	0.006	0.009	0.009	0.011	0.013
3-Omni-test	0.013	0.015	0.016	0.020	0.021	0.022
5-Omni-test	0.051	0.038	0.036	0.036	0.037	0.036
8-Omni-test	0.198	0.141	0.107	0.100	0.096	0.090
10-Omni-test	0.382	0.253	0.198	0.174	0.159	0.146
APS	1.600 (6)	0.667 (2)	0.467 (1)	0.667 (3)	1.000 (4)	1.267 (5)

(b) IGDX

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.036	0.036	0.034	0.036	0.037	0.036
SYM-PART1	0.071	0.064	0.064	4.285	3.992	1.763
SYM-PART2	0.082	0.074	0.187	2.607	3.050	4.882
SYM-PART3	0.073	0.065	0.108	0.638	1.527	1.942
SSUF1	0.094	0.104	0.113	0.118	0.128	0.140
SSUF3	0.030	0.043	0.046	0.056	0.066	0.072
3-Polygon	0.093	0.092	0.092	0.131	0.213	0.263
5-Polygon	0.136	0.138	0.143	0.186	0.218	0.227
8-Polygon	0.201	0.223	0.224	0.273	0.304	0.357
10-Polygon	0.187	0.195	0.201	0.220	0.246	0.271
2-Omni-test	0.025	0.024	0.033	0.896	0.906	0.609
3-Omni-test	0.085	0.106	0.826	0.847	0.626	1.144
5-Omni-test	1.384	1.404	1.503	1.756	1.788	1.890
8-Omni-test	3.357	3.151	3.169	3.222	3.287	3.376
10-Omni-test	4.392	4.155	4.102	4.174	4.175	4.226
APS	0.800 (2)	0.267 (1)	1.000 (3)	2.467 (4)	2.933 (5)	3.467 (6)

TABLE S.5: Results of eMOEA/D-ADA with the six L values on the 15 test problem instances. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.056	0.053	0.052	0.054	0.056	0.052
SYM-PART1	0.011	0.011	0.011	0.015	0.018	0.019
SYM-PART2	0.013	0.012	0.011	0.012	0.015	0.017
SYM-PART3	0.013	0.012	0.011	0.012	0.011	0.011
SSUF1	0.010	0.010	0.011	0.011	0.012	0.013
SSUF3	0.018	0.018	0.021	0.022	0.024	0.024
3-Polygon	0.040	0.040	0.040	0.040	0.040	0.040
5-Polygon	0.113	0.113	0.112	0.112	0.112	0.112
8-Polygon	0.168	0.171	0.168	0.168	0.167	0.167
10-Polygon	0.155	0.156	0.155	0.153	0.154	0.153
2-Omni-test	0.006	0.006	0.008	0.009	0.010	0.011
3-Omni-test	0.012	0.014	0.013	0.016	0.018	0.019
5-Omni-test	0.050	0.036	0.033	0.031	0.031	0.032
8-Omni-test	0.206	0.141	0.109	0.099	0.095	0.093
10-Omni-test	0.403	0.267	0.204	0.183	0.173	0.166
APS	1.667 (6)	0.867 (3)	0.467 (1)	0.533 (2)	0.867 (4)	1.067 (5)

(b) IGDX

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.040	0.040	0.040	0.042	0.044	0.044
SYM-PART1	0.072	0.065	0.067	2.971	2.928	1.420
SYM-PART2	0.083	0.074	0.220	2.861	3.164	3.770
SYM-PART3	0.072	0.063	0.101	0.538	1.392	2.170
SSUF1	0.098	0.105	0.116	0.121	0.131	0.148
SSUF3	0.035	0.038	0.061	0.070	0.075	0.083
3-Polygon	0.087	0.086	0.089	0.171	0.222	0.236
5-Polygon	0.177	0.184	0.195	0.236	0.289	0.342
8-Polygon	0.200	0.223	0.223	0.293	0.321	0.352
10-Polygon	0.180	0.197	0.200	0.255	0.236	0.270
2-Omni-test	0.025	0.024	0.031	0.887	0.838	0.587
3-Omni-test	0.084	0.103	0.667	0.734	0.641	1.074
5-Omni-test	1.398	1.404	1.493	1.671	1.688	1.761
8-Omni-test	3.349	3.192	3.175	3.194	3.229	3.299
10-Omni-test	4.377	4.193	4.101	4.122	4.137	4.213
APS	0.867 (2)	0.333 (1)	1.133 (3)	2.200 (4)	2.533 (5)	3.200 (6)

TABLE S.6: Results of NSGA-III-ADA with the six L values on the 15 test problem instances. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.021	0.021	0.021	0.020	0.020	0.020
SYM-PART1	0.012	0.011	0.011	0.011	0.011	0.011
SYM-PART2	0.013	0.012	0.012	0.012	0.012	0.012
SYM-PART3	0.014	0.012	0.012	0.012	0.012	0.012
SSUF1	0.004	0.004	0.004	0.004	0.004	0.004
SSUF3	0.013	0.013	0.014	0.015	0.017	0.020
3-Polygon	0.043	0.044	0.044	0.044	0.045	0.046
5-Polygon	0.042	0.040	0.040	0.041	0.044	0.044
8-Polygon	0.120	0.118	0.120	0.120	0.119	0.119
10-Polygon	0.127	0.126	0.127	0.124	0.124	0.122
2-Omni-test	0.006	0.006	0.006	0.006	0.006	0.006
3-Omni-test	0.012	0.011	0.011	0.011	0.011	0.011
5-Omni-test	0.048	0.038	0.032	0.031	0.029	0.030
8-Omni-test	0.212	0.148	0.110	0.099	0.095	0.091
10-Omni-test	0.423	0.267	0.202	0.173	0.159	0.153
APS	2.600 (6)	1.467 (5)	0.733 (4)	0.133 (1)	0.133 (1)	0.200 (3)

(b) IGDX

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.037	0.038	0.038	0.038	0.042	0.042
SYM-PART1	0.073	0.065	0.066	0.924	1.458	0.728
SYM-PART2	0.083	0.077	0.264	2.959	2.363	2.399
SYM-PART3	0.079	0.063	0.132	0.618	1.824	2.374
SSUF1	0.063	0.066	0.067	0.070	0.072	0.080
SSUF3	0.024	0.024	0.035	0.046	0.060	0.072
3-Polygon	0.096	0.096	0.097	0.507	0.393	0.374
5-Polygon	0.079	0.079	0.079	0.094	0.113	0.119
8-Polygon	0.136	0.136	0.143	0.181	0.240	0.284
10-Polygon	0.137	0.139	0.144	0.158	0.178	0.194
2-Omni-test	0.025	0.024	0.025	0.149	0.157	0.122
3-Omni-test	0.083	0.085	0.307	0.477	0.447	0.465
5-Omni-test	1.412	1.408	1.466	1.597	1.636	1.743
8-Omni-test	3.384	3.207	3.203	3.215	3.253	3.299
10-Omni-test	4.435	4.210	4.167	4.130	4.179	4.268
APS	1.000 (3)	0.067 (1)	0.533 (2)	2.200 (4)	2.733 (5)	3.533 (6)

TABLE S.7: Results of θ-DEA-ADA with the six L values on the 15 test problem instances. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.021	0.021	0.021	0.020	0.020	0.020
SYM-PART1	0.014	0.013	0.013	0.013	0.013	0.013
SYM-PART2	0.015	0.014	0.014	0.014	0.014	0.014
SYM-PART3	0.016	0.015	0.014	0.013	0.014	0.014
SSUF1	0.004	0.004	0.004	0.004	0.004	0.004
SSUF3	0.015	0.012	0.010	0.008	0.008	0.008
3-Polygon	0.040	0.040	0.040	0.040	0.040	0.038
5-Polygon	0.069	0.067	0.068	0.068	0.070	0.071
8-Polygon	0.129	0.128	0.128	0.129	0.129	0.127
10-Polygon	0.143	0.141	0.140	0.142	0.139	0.138
2-Omni-test	0.007	0.007	0.007	0.007	0.007	0.007
3-Omni-test	0.012	0.011	0.012	0.011	0.012	0.012
5-Omni-test	0.047	0.036	0.031	0.029	0.028	0.028
8-Omni-test	0.207	0.141	0.103	0.090	0.081	0.078
10-Omni-test	0.398	0.261	0.183	0.161	0.154	0.140
APS	2.200 (6)	1.267 (5)	0.400 (4)	0.200 (3)	0.000 (1)	0.000 (1)

(b) IGDX

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.036	0.037	0.037	0.038	0.039	0.042
SYM-PART1	0.072	0.064	0.063	0.840	1.453	0.760
SYM-PART2	0.080	0.073	0.262	2.666	2.672	2.168
SYM-PART3	0.073	0.064	0.167	0.614	1.498	2.208
SSUF1	0.062	0.065	0.066	0.070	0.072	0.075
SSUF3	0.023	0.024	0.026	0.027	0.030	0.043
3-Polygon	0.094	0.095	0.097	0.535	0.522	0.368
5-Polygon	0.085	0.085	0.093	0.143	0.157	0.188
8-Polygon	0.154	0.158	0.162	0.214	0.256	0.249
10-Polygon	0.159	0.164	0.171	0.184	0.209	0.214
2-Omni-test	0.024	0.023	0.023	0.115	0.195	0.145
3-Omni-test	0.081	0.084	0.242	0.400	0.378	0.499
5-Omni-test	1.396	1.386	1.408	1.618	1.606	1.706
8-Omni-test	3.371	3.194	3.135	3.176	3.217	3.280
10-Omni-test	4.404	4.178	4.116	4.134	4.147	4.212
APS	1.067 (3)	0.200 (1)	0.867 (2)	2.133 (4)	2.667 (5)	3.200 (6)

TABLE S.8: Results of RVEA-ADA with the six L values on the 15 test problem instances. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.069	0.048	0.065	0.063	0.062	0.059
SYM-PART1	0.022	0.017	0.022	0.024	0.025	0.025
SYM-PART2	0.024	0.019	0.022	0.024	0.026	0.027
SYM-PART3	0.025	0.020	0.023	0.025	0.026	0.027
SSUF1	0.016	0.006	0.017	0.018	0.017	0.016
SSUF3	0.024	0.014	0.028	0.030	0.031	0.029
3-Polygon	0.043	0.040	0.045	0.043	0.046	0.046
5-Polygon	0.115	0.108	0.113	0.112	0.114	0.120
8-Polygon	0.165	0.165	0.171	0.175	0.185	0.196
10-Polygon	0.164	0.170	0.170	0.177	0.182	0.189
2-Omni-test	0.011	0.008	0.012	0.012	0.013	0.013
3-Omni-test	0.027	0.021	0.029	0.030	0.029	0.029
5-Omni-test	0.095	0.077	0.083	0.084	0.086	0.081
8-Omni-test	0.296	0.234	0.228	0.211	0.218	0.207
10-Omni-test	0.514	0.388	0.347	0.334	0.325	0.327
APS	1.933 (6)	0.467 (1)	1.067 (3)	0.933 (2)	1.333 (4)	1.867 (5)

(b) IGDX

	$L = \lfloor 0.05\mu \rfloor$	$L = \lfloor 0.1\mu \rfloor$	$L = \lfloor 0.2\mu \rfloor$	$L = \lfloor 0.3\mu \rfloor$	$L = \lfloor 0.4\mu \rfloor$	$L = \lfloor 0.5\mu \rfloor$
Two-On-One	0.049	0.066	0.051	0.051	0.052	0.053
SYM-PART1	0.092	0.085	0.094	2.442	1.905	1.048
SYM-PART2	0.097	0.093	0.674	3.926	3.813	4.014
SYM-PART3	0.107	0.078	0.211	1.241	2.372	2.745
SSUF1	0.129	0.077	0.143	0.144	0.141	0.148
SSUF3	0.041	0.028	0.067	0.083	0.083	0.086
3-Polygon	0.090	0.090	0.090	1.693	1.797	1.495
5-Polygon	0.109	0.100	0.130	1.778	1.800	2.035
8-Polygon	0.149	0.156	0.196	1.809	1.882	2.032
10-Polygon	0.131	0.139	0.184	1.599	1.573	1.483
2-Omni-test	0.032	0.030	0.035	0.677	0.671	0.369
3-Omni-test	0.105	0.135	0.665	0.784	0.602	0.805
5-Omni-test	1.600	1.678	1.867	1.995	2.014	2.093
8-Omni-test	3.524	3.483	3.603	3.667	3.735	3.749
10-Omni-test	4.531	4.455	4.563	4.650	4.695	4.668
APS	0.467 (1)	0.600 (2)	1.600 (3)	2.667 (4)	2.933 (5)	3.067 (6)

TABLE S.9: Results of the six EMOAs and their ADA versions on the 21 test problems with distance-related variables. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. "Orig." and "ADA" represent an original EMOA and its ADA version, respectively. HPS2, ..., MMMOP6C are two-objective problems, and MMMOP1B, ..., MMMOP6D are three-objective problems. The best results between them are shaded. The symbols $+$, $-$, and $\approx$ indicate that a given ADA version performs significantly better ($+$), significantly worse ($-$), and not significantly better or worse ($\approx$) compared to its original EMOA according to the Wilcoxon rank-sum test with $p < 0.001$.

(a) IGD$^+$

	MOEA/D-AGR		MOEA/D-DU		eMOEA/D		NSGA-III		θ-DEA		RVEA	
	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA
HPS2	0.007	19.543−	0.006	19.572−	0.083	22.934−	0.004	32.527−	0.003	37.761−	0.018	15.494−
MMMOP1A	0.004	0.013−	0.003	0.012−	0.024	0.009≈	0.003	0.050−	0.003	0.059−	0.003	0.021−
MMMOP2A	0.0001	0.0002−	0.0001	0.0001−	0.0003	0.0001+	0.0008	0.0081−	0.0010	0.0002+	0.0001	0.0007−
MMMOP3A	0.002	0.002−	0.002	0.002−	0.002	0.002−	0.002	0.002−	0.002	0.003−	0.002	0.007−
MMMOP3C	0.002	0.012−	0.002	0.011−	0.003	0.009−	0.002	0.012−	0.002	0.010−	0.003	0.040−
MMMOP4A	0.003	0.005−	0.002	0.005−	0.013	0.004≈	0.002	0.009−	0.002	0.010−	0.002	0.009−
MMMOP4C	0.004	8.512−	0.003	7.887−	0.015	8.585−	0.003	9.547−	0.003	10.544−	0.004	8.113−
MMMOP5A	0.002	0.006−	0.002	0.005−	0.014	0.005≈	0.002	0.008−	0.002	0.016−	0.002	0.010−
MMMOP5C	0.004	7.884−	0.003	8.581−	0.027	8.341−	0.003	10.928−	0.003	9.901−	0.006	8.310−
MMMOP6A	0.002	0.005−	0.002	0.005−	0.005	0.005+	0.003	0.004−	0.002	0.003−	0.005	0.007−
MMMOP6C	0.005	0.014−	0.002	0.013−	0.034	0.013≈	0.004	0.029−	0.002	0.031−	0.007	0.034−
MMMOP1B	0.048	3.593−	0.029	3.395−	0.036	1.978−	0.033	5.223−	0.032	4.523−	0.035	2.301−
MMMOP2B	0.003	0.010−	0.001	0.008−	0.001	0.005−	0.002	0.104−	0.001	0.013−	0.001	0.015−
MMMOP3B	0.032	0.040−	0.021	0.026−	0.027	0.030−	0.021	0.037−	0.021	0.029−	0.021	0.042−
MMMOP3D	0.032	0.062−	0.021	0.047−	0.026	0.037−	0.022	0.057−	0.022	0.040−	0.021	0.078−
MMMOP4B	0.039	16.555−	0.027	16.533−	0.037	7.900−	0.032	20.661−	0.031	16.865−	0.032	12.095−
MMMOP4D	0.041	11.018−	0.026	10.364−	0.042	5.082−	0.028	14.278−	0.029	11.089−	0.029	7.311−
MMMOP5B	0.040	16.543−	0.023	15.878−	0.037	8.651−	0.029	20.789−	0.028	17.076−	0.029	12.518−
MMMOP5D	0.036	10.908−	0.024	11.148−	0.040	5.029−	0.029	13.000−	0.026	9.812−	0.027	7.154−
MMMOP6B	0.031	0.048−	0.022	0.035−	0.038	0.038≈	0.033	0.047−	0.028	0.036−	0.035	0.049−
MMMOP6D	0.030	0.065−	0.022	0.053−	0.037	0.045−	0.027	0.070−	0.026	0.065−	0.031	0.082−
APS	0.0	1.0	0.0	1.0	0.095	0.667	0.0	1.0	0.048	0.952	0.0	1.0

(b) IGDX

	MOEA/D-AGR		MOEA/D-DU		eMOEA/D		NSGA-III		θ-DEA		RVEA	
	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA	Orig.	ADA
HPS2	0.257	0.362−	0.219	0.371−	0.250	0.403−	0.206	0.416−	0.180	0.421−	0.188	0.362−
MMMOP1A	0.298	0.063+	0.241	0.061+	0.258	0.055+	0.290	0.112+	0.286	0.110+	0.246	0.076+
MMMOP2A	0.650	0.239+	0.585	0.236+	0.637	0.218+	0.586	0.262+	0.554	0.250+	0.627	0.300+
MMMOP3A	0.011	0.009≈	0.010	0.008≈	0.014	0.009≈	0.008	0.009−	0.013	0.009≈	0.009	0.011−
MMMOP3C	0.245	0.071+	0.188	0.073+	0.261	0.064+	0.261	0.076+	0.259	0.070+	0.234	0.137+
MMMOP4A	0.047	0.004+	0.018	0.003+	0.070	0.003+	0.026	0.005+	0.030	0.005+	0.023	0.006+
MMMOP4C	0.271	0.308−	0.248	0.290−	0.289	0.292≈	0.271	0.320−	0.259	0.297≈	0.248	0.304−
MMMOP5A	0.035	0.005+	0.016	0.005+	0.068	0.005+	0.014	0.006+	0.013	0.006+	0.010	0.009≈
MMMOP5C	0.284	0.297≈	0.267	0.283≈	0.292	0.293≈	0.272	0.326−	0.272	0.336−	0.250	0.299≈
MMMOP6A	0.085	0.014+	0.044	0.014+	0.142	0.014+	0.057	0.012+	0.032	0.010+	0.037	0.020+
MMMOP6C	0.489	0.129+	0.389	0.125+	0.413	0.131+	0.435	0.181+	0.431	0.176+	0.418	0.181+
MMMOP1B	0.350	0.341≈	0.298	0.318≈	0.317	0.281≈	0.323	0.374−	0.315	0.342≈	0.321	0.295≈
MMMOP2B	0.722	0.375+	0.558	0.338+	0.662	0.323+	0.575	0.416+	0.562	0.345+	0.691	0.405+
MMMOP3B	0.057	0.104−	0.049	0.078−	0.053	0.080−	0.052	0.112−	0.051	0.088−	0.050	0.124−
MMMOP3D	0.275	0.186+	0.192	0.179≈	0.215	0.117+	0.207	0.190≈	0.193	0.157≈	0.197	0.201≈
MMMOP4B	0.141	0.356−	0.105	0.352−	0.124	0.283−	0.129	0.404−	0.114	0.371−	0.098	0.328−
MMMOP4D	0.343	0.357≈	0.283	0.345−	0.322	0.275+	0.309	0.377−	0.316	0.350−	0.278	0.293≈
MMMOP5B	0.171	0.374−	0.116	0.354−	0.136	0.295−	0.146	0.392−	0.129	0.360−	0.117	0.322−
MMMOP5D	0.358	0.368≈	0.296	0.351−	0.337	0.286+	0.335	0.383−	0.315	0.345≈	0.292	0.305≈
MMMOP6B	0.319	0.152+	0.215	0.144+	0.243	0.135+	0.240	0.155+	0.213	0.148+	0.193	0.175+
MMMOP6D	0.550	0.205+	0.417	0.200+	0.353	0.183+	0.453	0.215+	0.455	0.216+	0.438	0.261+
APS	0.524	0.238	0.476	0.333	0.619	0.190	0.476	0.476	0.476	0.286	0.429	0.286

TABLE S.10: Results of the nine EMMAs (the six ADA-based algorithms, TriMOEA-TA&R, MO_Ring_PSO_SCD, and Omni-optimizer) on the 21 test problem instances with distance-related variables. HPS2, ..., MMMOP6C are two-objective problems, and MMMOP1B, ..., MMMOP6D are three-objective problems. AGR, DU, TriMOEA, and MO_Ring stand for MOEA/D-AGR, MOEA/D-DU, TriMOEA-TA&R, and MO_Ring_PSO_SCD, respectively. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the six configurations are reported at the bottom of Tables (a) and (b).

(a) IGD$^+$

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	TriMOEA	MO_Ring	Omni-optimizer
HPS2	19.543	19.572	22.934	32.527	37.761	15.494	0.006	35.600	0.071
MMMOP1A	0.013	0.012	0.009	0.050	0.059	0.021	0.003	0.237	0.023
MMMOP2A	0.0002	0.0001	0.0001	0.0081	0.0002	0.0007	0.0014	0.0008	0.0004
MMMOP3A	0.002	0.002	0.002	0.002	0.003	0.007	0.002	0.004	0.003
MMMOP3C	0.012	0.011	0.009	0.012	0.010	0.040	0.002	0.039	0.004
MMMOP4A	0.005	0.005	0.004	0.009	0.010	0.009	0.002	0.008	0.010
MMMOP4C	8.512	7.887	8.585	9.547	10.544	8.113	0.004	33.241	0.029
MMMOP5A	0.006	0.005	0.005	0.008	0.016	0.010	0.002	0.010	0.011
MMMOP5C	7.884	8.581	8.341	10.928	9.901	8.310	0.003	33.317	0.033
MMMOP6A	0.005	0.005	0.005	0.004	0.003	0.007	0.007	0.004	0.003
MMMOP6C	0.014	0.013	0.013	0.029	0.031	0.034	0.055	0.062	0.011
MMMOP1B	3.593	3.395	1.978	5.223	4.523	2.301	0.028	30.772	0.107
MMMOP2B	0.010	0.008	0.005	0.104	0.013	0.015	0.003	0.016	0.005
MMMOP3B	0.040	0.026	0.030	0.037	0.029	0.042	0.023	0.096	0.042
MMMOP3D	0.062	0.047	0.037	0.057	0.040	0.078	0.023	0.121	0.043
MMMOP4B	16.555	16.533	7.900	20.661	16.865	12.095	0.032	42.979	0.086
MMMOP4D	11.018	10.364	5.082	14.278	11.089	7.311	0.030	34.145	0.081
MMMOP5B	16.543	15.878	8.651	20.789	17.076	12.518	0.027	40.522	0.083
MMMOP5D	10.908	11.148	5.029	13.000	9.812	7.154	0.026	36.889	0.082
MMMOP6B	0.048	0.035	0.038	0.047	0.036	0.049	0.029	0.052	0.044
MMMOP6D	0.065	0.053	0.045	0.070	0.065	0.082	0.028	0.144	0.052
APS	2.952 (5)	2.190 (3)	1.476 (2)	4.190 (7)	3.048 (6)	4.381 (8)	1.048 (1)	6.810 (9)	2.286 (4)

(b) IGDX

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	TriMOEA	MO_Ring	Omni-optimizer
HPS2	0.362	0.371	0.403	0.416	0.421	0.362	0.144	0.481	0.089
MMMOP1A	0.063	0.061	0.055	0.112	0.110	0.076	0.014	0.204	0.208
MMMOP2A	0.239	0.236	0.218	0.262	0.250	0.300	0.291	0.372	0.462
MMMOP3A	0.009	0.008	0.009	0.009	0.009	0.011	0.012	0.015	0.014
MMMOP3C	0.071	0.073	0.064	0.076	0.070	0.137	0.107	0.170	0.181
MMMOP4A	0.004	0.003	0.003	0.005	0.005	0.006	0.030	0.012	0.012
MMMOP4C	0.308	0.290	0.292	0.320	0.297	0.304	0.088	0.404	0.240
MMMOP5A	0.005	0.005	0.005	0.006	0.006	0.009	0.010	0.011	0.012
MMMOP5C	0.297	0.283	0.293	0.326	0.336	0.299	0.068	0.385	0.234
MMMOP6A	0.014	0.014	0.014	0.012	0.010	0.020	0.041	0.015	0.021
MMMOP6C	0.129	0.125	0.131	0.181	0.176	0.181	0.298	0.320	0.386
MMMOP1B	0.341	0.318	0.281	0.374	0.342	0.295	0.113	0.446	0.248
MMMOP2B	0.375	0.338	0.323	0.416	0.345	0.405	0.349	0.486	0.531
MMMOP3B	0.104	0.078	0.080	0.112	0.088	0.124	0.165	0.241	0.129
MMMOP3D	0.186	0.179	0.117	0.190	0.157	0.201	0.183	0.275	0.180
MMMOP4B	0.356	0.352	0.283	0.404	0.371	0.328	0.101	0.415	0.074
MMMOP4D	0.357	0.345	0.275	0.377	0.350	0.293	0.123	0.414	0.241
MMMOP5B	0.374	0.354	0.295	0.392	0.360	0.322	0.098	0.412	0.075
MMMOP5D	0.368	0.351	0.286	0.383	0.345	0.305	0.109	0.425	0.260
MMMOP6B	0.152	0.144	0.135	0.155	0.148	0.175	0.205	0.168	0.194
MMMOP6D	0.205	0.200	0.183	0.215	0.216	0.261	0.243	0.346	0.385
APS	2.000 (3)	1.476 (2)	0.857 (1)	3.000 (6)	2.190 (4)	3.667 (7)	2.667 (5)	6.905 (9)	4.143 (8)

TABLE S.11: Comparison with the three EMMAs with the UEA on the 15 test problem instances. AGR, DU, U-TriMOEA, U-MO_Ring, and U-Omni-optimizer stand for MOEA/D-AGR, MOEA/D-DU, UEA-TriMOEA-TA&R, UEA-MO_Ring_PSO_SCD, and UEA-Omni-optimizer, respectively. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the algorithms are reported at the bottom of Tables (a) and (b). Since IGD$^+$ and IGDX cannot assess solution sets with different sizes as described in Subsection III-H, we calculate the IGD$^+$ and IGDX values using only N non-dominated solutions selected from the UEA. We use two post-processing methods as in ADA. For IGDX, we use the solution distance-based selection method in ADA. For IGD$^+$, we use the selection method in NSGA-III-ADA.

(a) IGD$^+$

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	U-TriMOEA	U-MO_Ring	U-Omni-optimizer
Two-On-One	0.051	0.046	0.053	0.021	0.021	0.048	0.020	0.021	0.019
SYM-PART1	0.011	0.011	0.011	0.011	0.013	0.017	0.011	0.014	0.011
SYM-PART2	0.013	0.011	0.012	0.012	0.014	0.019	0.012	0.016	0.011
SYM-PART3	0.013	0.012	0.012	0.012	0.015	0.020	0.012	0.016	0.011
SSUF1	0.011	0.010	0.010	0.004	0.004	0.006	0.003	0.003	0.003
SSUF3	0.018	0.017	0.018	0.013	0.012	0.014	0.013	0.009	0.009
3-Polygon	0.036	0.040	0.040	0.044	0.040	0.040	0.040	0.040	0.040
5-Polygon	0.053	0.104	0.113	0.040	0.067	0.108	0.033	0.033	0.033
8-Polygon	0.110	0.170	0.171	0.118	0.128	0.165	0.112	0.112	0.111
10-Polygon	0.089	0.162	0.156	0.126	0.141	0.170	0.170	0.170	0.169
2-Omni-test	0.006	0.006	0.006	0.006	0.007	0.008	0.006	0.007	0.006
3-Omni-test	0.017	0.015	0.014	0.011	0.011	0.021	0.011	0.018	0.009
5-Omni-test	0.045	0.038	0.036	0.038	0.036	0.077	0.036	0.150	0.019
8-Omni-test	0.147	0.141	0.141	0.148	0.141	0.234	0.134	0.679	0.038
10-Omni-test	0.270	0.253	0.267	0.267	0.261	0.388	0.249	1.231	0.057
APS	3.800 (7)	2.800 (4)	3.133 (6)	2.467 (3)	3.000 (5)	5.933 (9)	1.333 (2)	4.467 (8)	0.533 (1)

(b) IGDX

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	U-TriMOEA	U-MO_Ring	U-Omni-optimizer
Two-On-One	0.041	0.036	0.040	0.038	0.037	0.066	0.408	0.029	0.024
SYM-PART1	0.071	0.064	0.065	0.065	0.064	0.085	1.048	0.121	0.138
SYM-PART2	0.082	0.074	0.074	0.077	0.073	0.093	9.247	0.109	0.715
SYM-PART3	0.068	0.065	0.063	0.063	0.064	0.078	4.549	0.556	1.100
SSUF1	0.105	0.104	0.105	0.066	0.065	0.077	0.075	0.058	0.058
SSUF3	0.048	0.043	0.038	0.024	0.024	0.028	0.084	0.016	0.052
3-Polygon	0.090	0.092	0.086	0.096	0.095	0.090	2.224	0.105	0.156
5-Polygon	0.077	0.138	0.184	0.079	0.085	0.100	2.455	0.091	0.098
8-Polygon	0.116	0.223	0.223	0.136	0.158	0.156	2.581	0.116	0.138
10-Polygon	0.079	0.195	0.197	0.139	0.164	0.139	2.141	0.095	0.101
2-Omni-test	0.026	0.024	0.024	0.024	0.023	0.030	2.425	0.028	0.021
3-Omni-test	0.115	0.106	0.103	0.085	0.084	0.135	3.673	0.273	0.128
5-Omni-test	1.402	1.404	1.404	1.408	1.386	1.678	5.092	2.204	1.571
8-Omni-test	3.160	3.151	3.192	3.207	3.194	3.483	6.758	4.145	3.409
10-Omni-test	4.175	4.155	4.193	4.210	4.178	4.455	7.627	5.169	4.410
APS	2.267 (3)	2.400 (4)	2.467 (5)	1.200 (1)	1.400 (2)	4.533 (8)	7.733 (9)	4.133 (7)	2.600 (6)

TABLE S.12: Comparison with the three EMMAs with the UEA on the 21 test problems with distance-related variables. HPS2, ..., MMMOP6C are two-objective problems, and MMMOP1B, ..., MMMOP6D are three-objective problems. AGR, DU, U-TriMOEA, U-MO_Ring, and U-Omni-optimizer stand for MOEA/D-AGR, MOEA/D-DU, UEA-TriMOEA-TA&R, UEA-MO_Ring_PSO_SCD, and UEA-Omni-optimizer, respectively. Tables (a) and (b) show the mean IGD$^+$ and IGDX values, respectively. The best results between the competitors are shaded. The APS values of the algorithms are reported at the bottom of Tables (a) and (b). Since IGD$^+$ and IGDX cannot assess solution sets with different sizes as described in Subsection III-H, we calculate the IGD$^+$ and IGDX values using only N non-dominated solutions selected from the UEA. We use two post-processing methods as in ADA. For IGDX, we use the solution distance-based selection method in ADA. For IGD$^+$, we use the selection method in NSGA-III-ADA.

(a) IGD$^+$

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	U-TriMOEA	U-MO_Ring	U-Omni-optimizer
HPS2	19.543	19.572	22.934	32.527	37.761	15.494	0.057	35.592	0.012
MMMOP1A	0.013	0.012	0.009	0.050	0.059	0.021	0.003	0.236	0.004
MMMOP2A	0.0002	0.0001	0.0001	0.0081	0.0002	0.0007	0.0004	0.0007	0.0003
MMMOP3A	0.002	0.002	0.002	0.002	0.003	0.007	0.002	0.002	0.002
MMMOP3C	0.012	0.011	0.009	0.012	0.010	0.040	0.002	0.036	0.002
MMMOP4A	0.005	0.005	0.004	0.009	0.010	0.009	0.003	0.007	0.002
MMMOP4C	8.512	7.887	8.585	9.547	10.544	8.113	0.004	33.241	0.008
MMMOP5A	0.006	0.005	0.005	0.008	0.016	0.010	0.002	0.008	0.002
MMMOP5C	7.884	8.581	8.341	10.928	9.901	8.310	0.004	33.317	0.008
MMMOP6A	0.005	0.005	0.005	0.004	0.003	0.007	0.002	0.003	0.002
MMMOP6C	0.014	0.013	0.013	0.029	0.031	0.034	0.005	0.059	0.003
MMMOP1B	3.593	3.395	1.978	5.223	4.523	2.301	0.035	29.831	0.064
MMMOP2B	0.010	0.008	0.005	0.104	0.013	0.015	0.003	0.013	0.002
MMMOP3B	0.040	0.026	0.030	0.037	0.029	0.042	0.023	0.066	0.028
MMMOP3D	0.062	0.047	0.037	0.057	0.040	0.078	0.024	0.086	0.029
MMMOP4B	16.555	16.533	7.900	20.661	16.865	12.095	0.099	41.158	0.050
MMMOP4D	11.018	10.364	5.082	14.278	11.089	7.311	0.039	33.318	0.047
MMMOP5B	16.543	15.878	8.651	20.789	17.076	12.518	0.032	40.361	0.048
MMMOP5D	10.908	11.148	5.029	13.000	9.812	7.154	0.032	36.609	0.050
MMMOP6B	0.048	0.035	0.038	0.047	0.036	0.049	0.028	0.043	0.032
MMMOP6D	0.065	0.053	0.045	0.070	0.065	0.082	0.027	0.139	0.036
APS	3.333 (5)	2.619 (4)	2.143 (3)	4.619 (7)	3.381 (6)	4.857 (8)	0.381 (1)	6.524 (9)	0.905 (2)

(b) IGDX

	AGR-ADA	DU-ADA	eMOEA/D-ADA	NSGA-III-ADA	θ-DEA-ADA	RVEA-ADA	U-TriMOEA	U-MO_Ring	U-Omni-optimizer
HPS2	0.362	0.371	0.403	0.416	0.421	0.362	0.125	0.480	0.061
MMMOP1A	0.063	0.061	0.055	0.112	0.110	0.076	0.036	0.205	0.115
MMMOP2A	0.239	0.236	0.218	0.262	0.250	0.300	0.388	0.376	0.418
MMMOP3A	0.009	0.008	0.009	0.009	0.009	0.011	0.007	0.010	0.008
MMMOP3C	0.071	0.073	0.064	0.076	0.070	0.137	0.030	0.157	0.074
MMMOP4A	0.004	0.003	0.003	0.005	0.005	0.006	0.006	0.008	0.003
MMMOP4C	0.308	0.290	0.292	0.320	0.297	0.304	0.179	0.405	0.208
MMMOP5A	0.005	0.005	0.005	0.006	0.006	0.009	0.003	0.009	0.003
MMMOP5C	0.297	0.283	0.293	0.326	0.336	0.299	0.201	0.385	0.215
MMMOP6A	0.014	0.014	0.014	0.012	0.010	0.020	0.016	0.009	0.008
MMMOP6C	0.129	0.125	0.131	0.181	0.176	0.181	0.373	0.319	0.352
MMMOP1B	0.341	0.318	0.281	0.374	0.342	0.295	0.160	0.436	0.210
MMMOP2B	0.375	0.338	0.323	0.416	0.345	0.405	0.361	0.469	0.448
MMMOP3B	0.104	0.078	0.080	0.112	0.088	0.124	0.095	0.257	0.124
MMMOP3D	0.186	0.179	0.117	0.190	0.157	0.201	0.104	0.275	0.130
MMMOP4B	0.356	0.352	0.283	0.404	0.371	0.328	0.087	0.405	0.045
MMMOP4D	0.357	0.345	0.275	0.377	0.350	0.293	0.174	0.403	0.190
MMMOP5B	0.374	0.354	0.295	0.392	0.360	0.322	0.069	0.402	0.046
MMMOP5D	0.368	0.351	0.286	0.383	0.345	0.305	0.166	0.413	0.210
MMMOP6B	0.152	0.144	0.135	0.155	0.148	0.175	0.171	0.134	0.134
MMMOP6D	0.205	0.200	0.183	0.215	0.216	0.261	0.386	0.341	0.325
APS	2.619 (5)	2.048 (4)	1.333 (1)	3.619 (7)	2.714 (6)	4.286 (8)	1.714 (2)	6.667 (9)	1.810 (3)