A Supplementary File for
“An Analysis of Control Parameters of MOEA/D Under Two Different Optimization Scenarios”

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Abstract
This is a supplementary file for “An Analysis of Control Parameters of MOEA/D Under Two Different Optimization Scenarios”.

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Preprint submitted to Applied Soft Computing March 5, 2018
Figure S.1: Performance of MOEA/D with various $\mu$ settings on the DTLZ1 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.
Figure S.2: Performance of MOEA/D with various $\mu$ settings on the DTLZ2 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.
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Figure S.54: Influence of $\mu$ on the performance of MOEA/D using $g^{ pathology}$ with various $\theta$ values on the DTLZ2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.
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Figure S.84: Comparison of the two Chebyshev functions ($g^{chm}$ and $g^{chd}$) and $g^{pbi}$ with various $\theta$ values on the WFG2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50000 evaluations among 31 runs is shown.
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