

A Supplementary File for
“An Analysis of Control Parameters of MOEA/D
Under Two Different Optimization Scenarios”

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Abstract

This is a supplementary file for “An Analysis of Control Parameters of MOEA/D Under Two Different Optimization Scenarios”.

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(Hisao Ishibuchi)

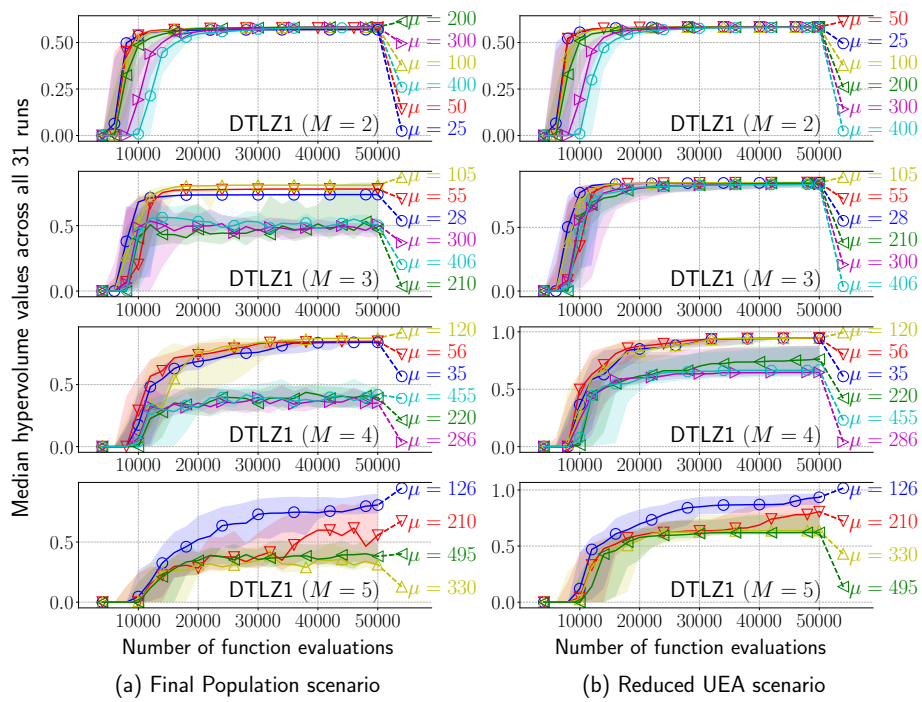


Figure S.1: Performance of MOEA/D with various μ settings on the DTLZ1 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

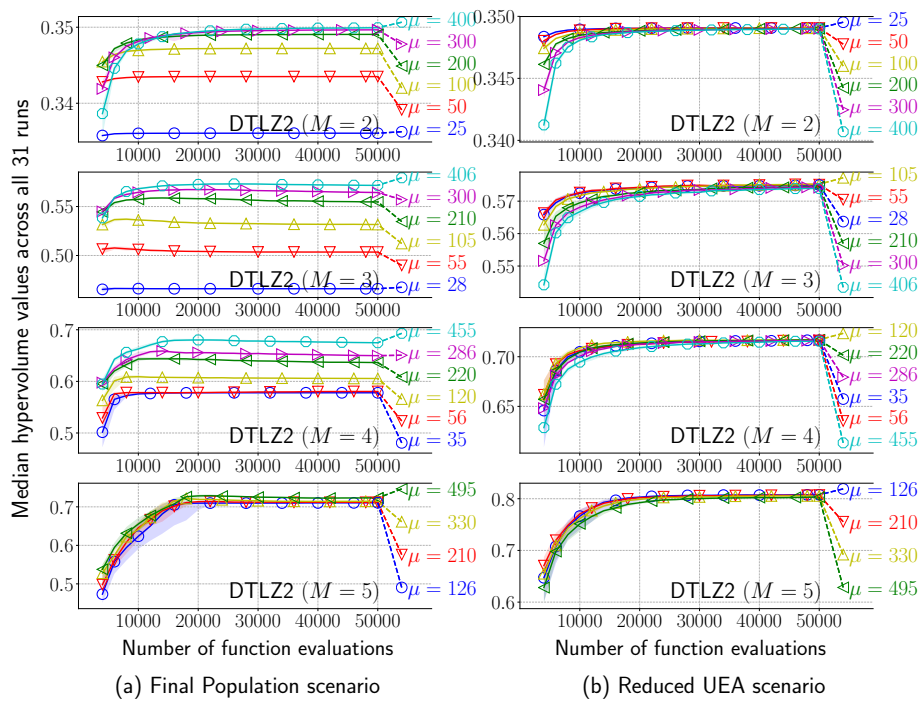


Figure S.2: Performance of MOEA/D with various μ settings on the DTLZ2 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

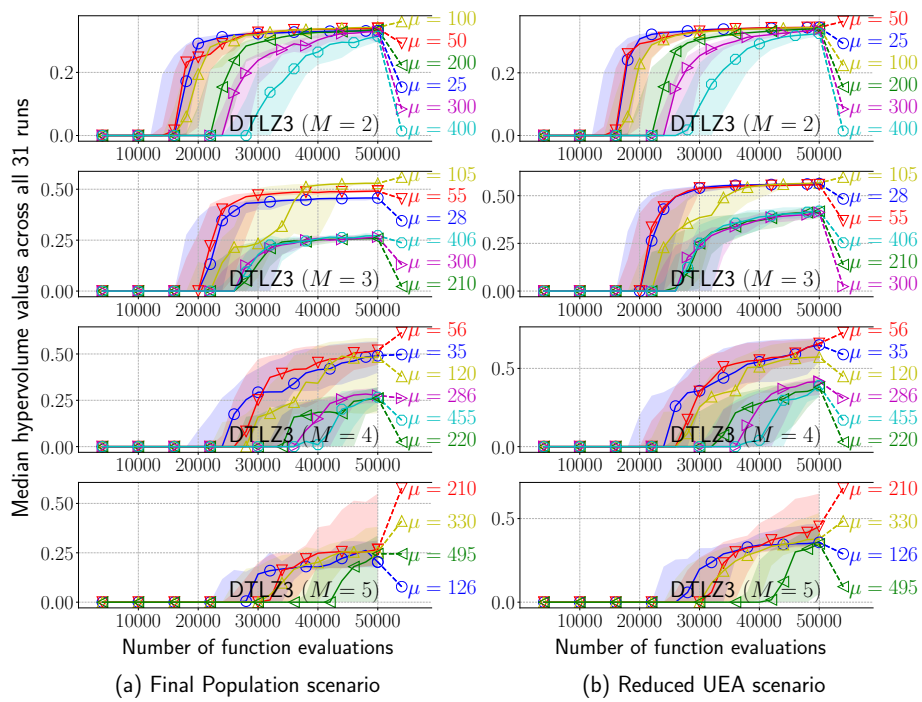


Figure S.3: Performance of MOEA/D with various μ settings on the DTLZ3 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

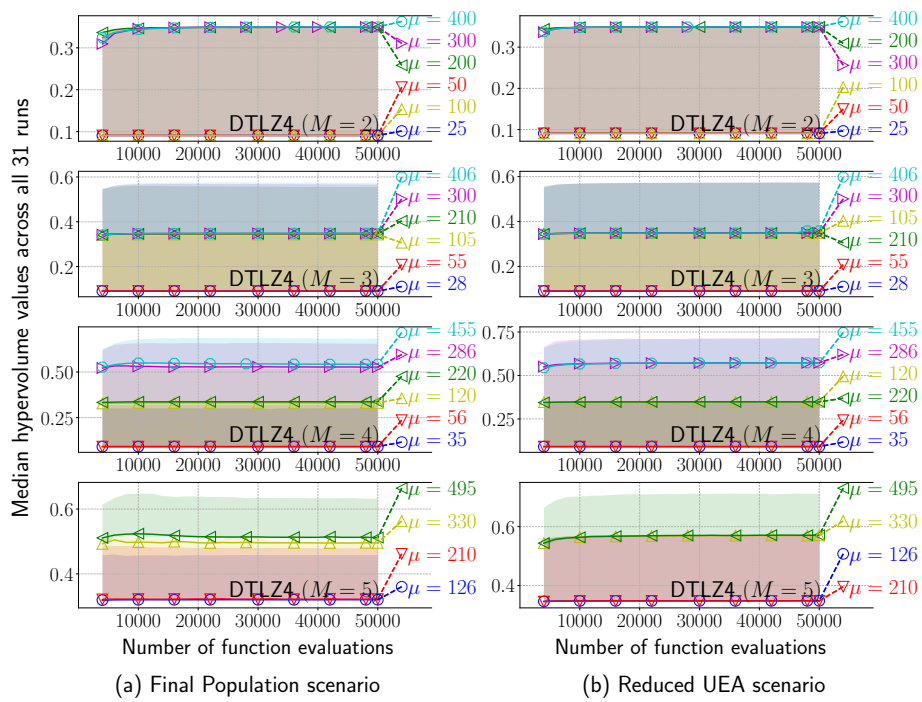


Figure S.4: Performance of MOEA/D with various μ settings on the DTLZ4 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

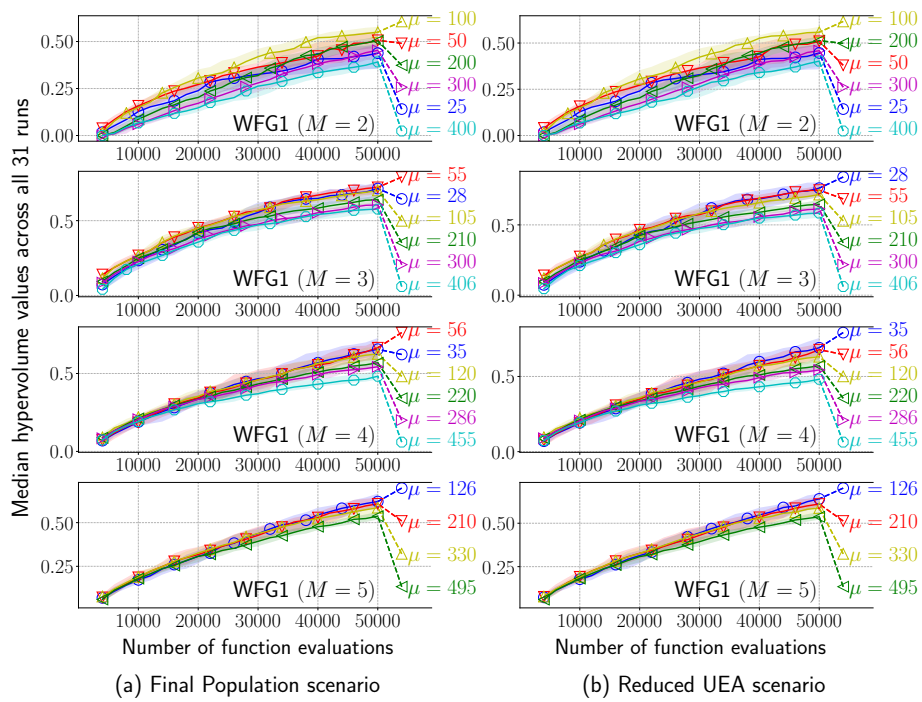


Figure S.5: Performance of MOEA/D with various μ settings on the WFG1 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

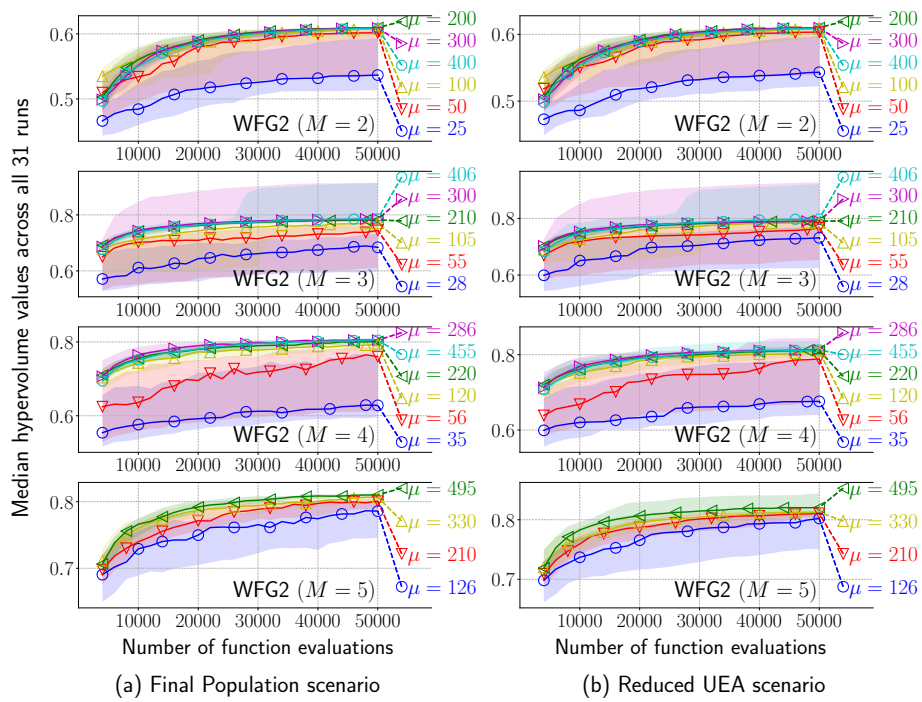


Figure S.6: Performance of MOEA/D with various μ settings on the WFG2 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

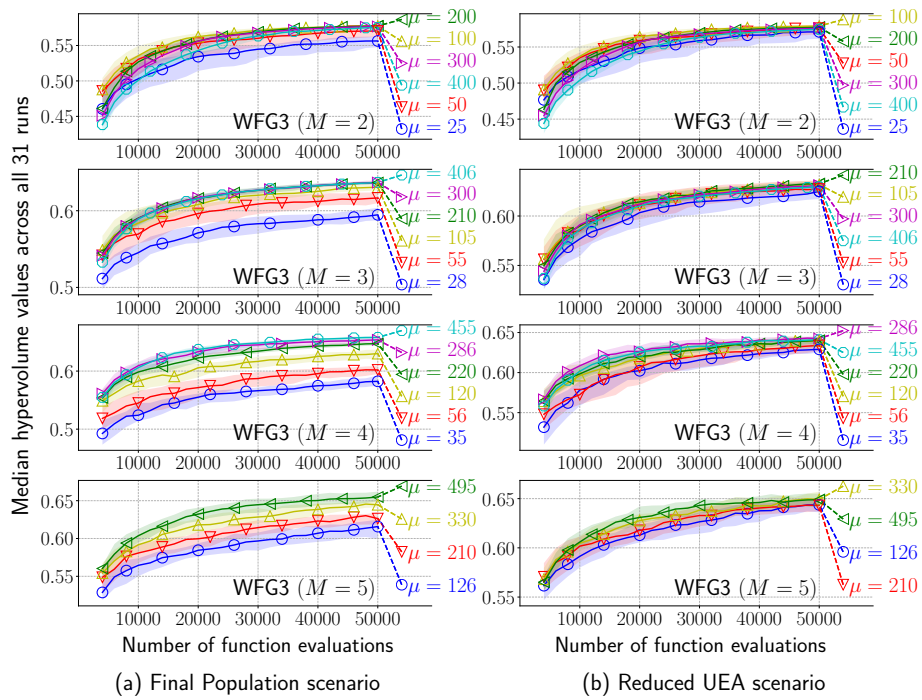


Figure S.7: Performance of MOEA/D with various μ settings on the WFG3 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

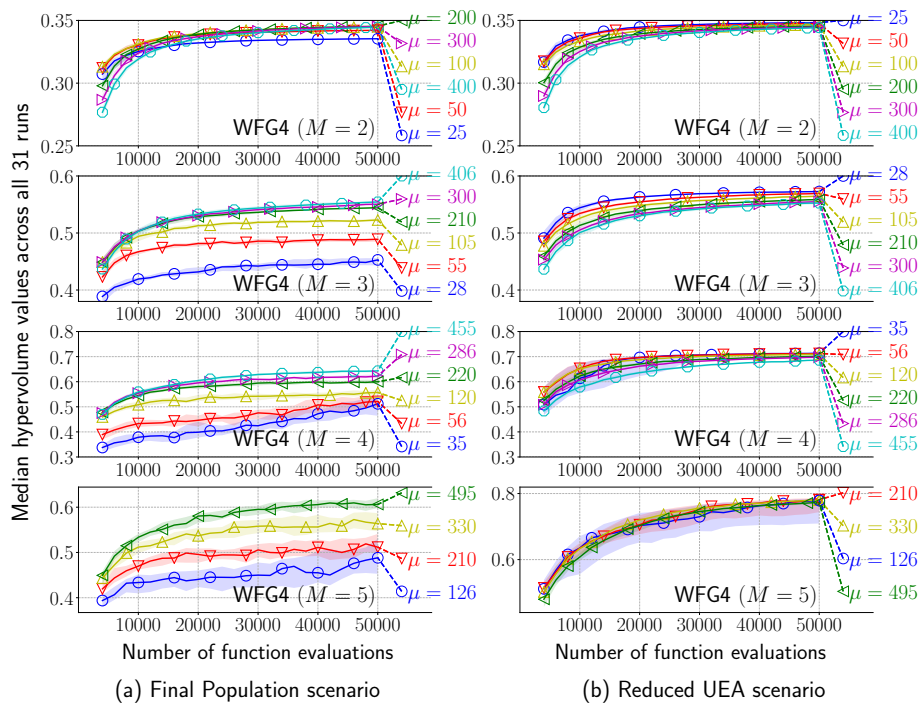


Figure S.8: Performance of MOEA/D with various μ settings on the WFG4 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

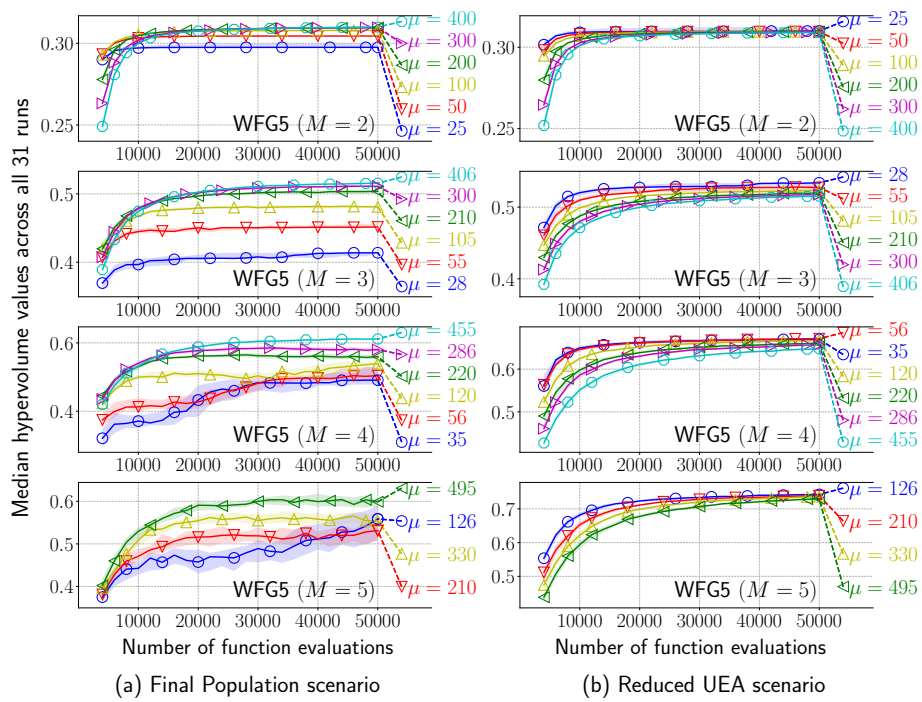


Figure S.9: Performance of MOEA/D with various μ settings on the WFG5 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

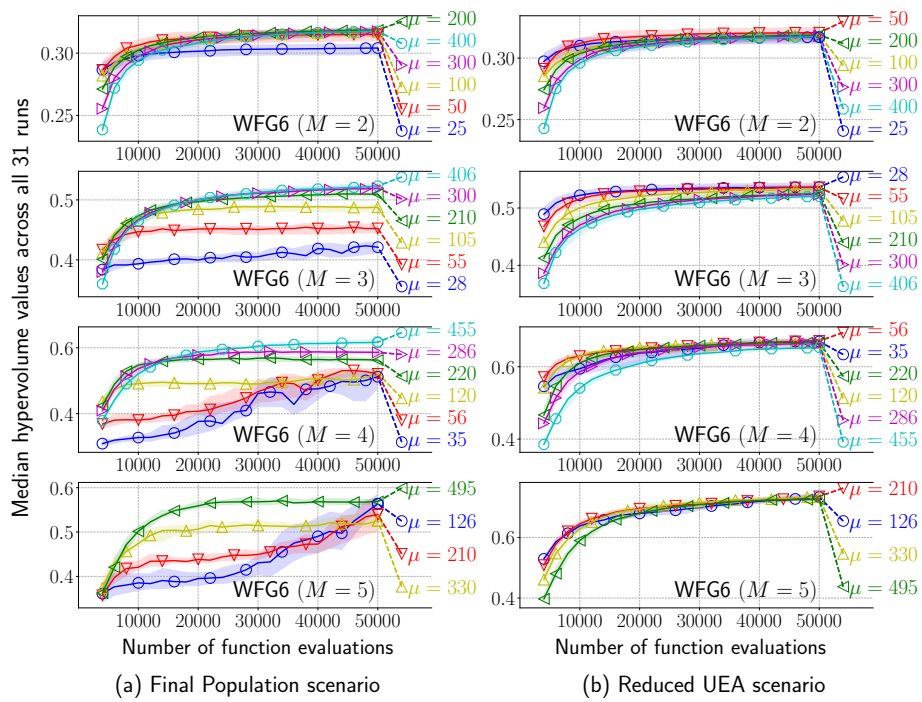


Figure S.10: Performance of MOEA/D with various μ settings on the WFG6 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

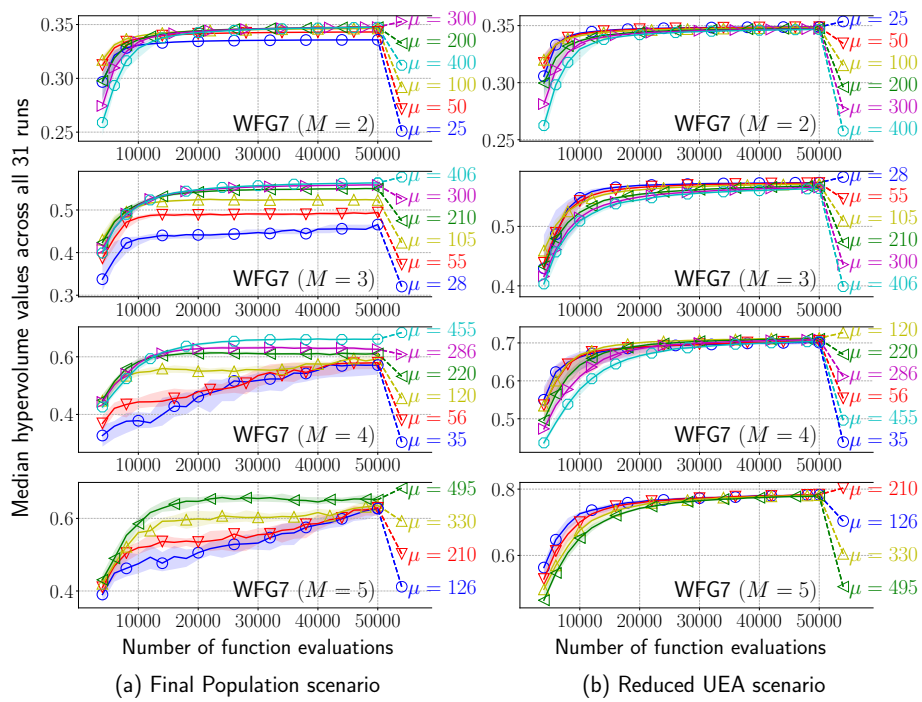


Figure S.11: Performance of MOEA/D with various μ settings on the WFG7 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

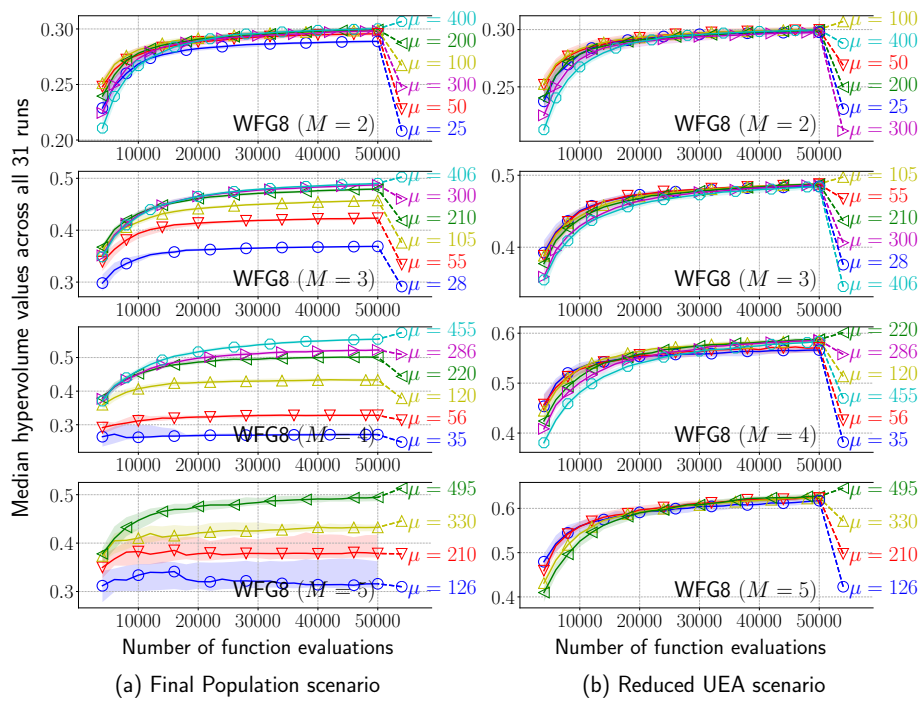


Figure S.12: Performance of MOEA/D with various μ settings on the WFG8 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

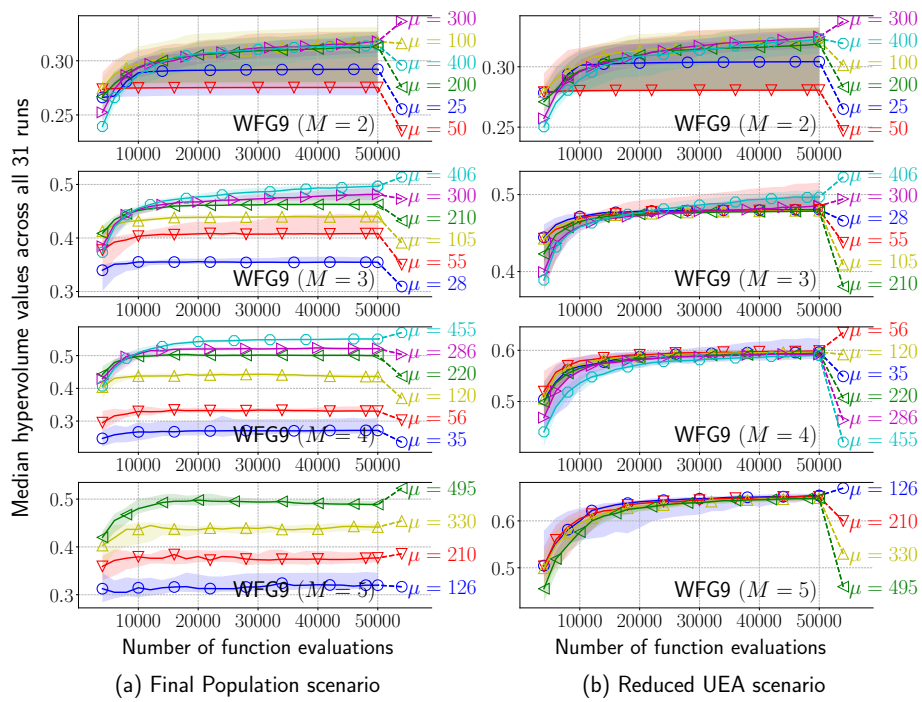


Figure S.13: Performance of MOEA/D with various μ settings on the WFG9 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

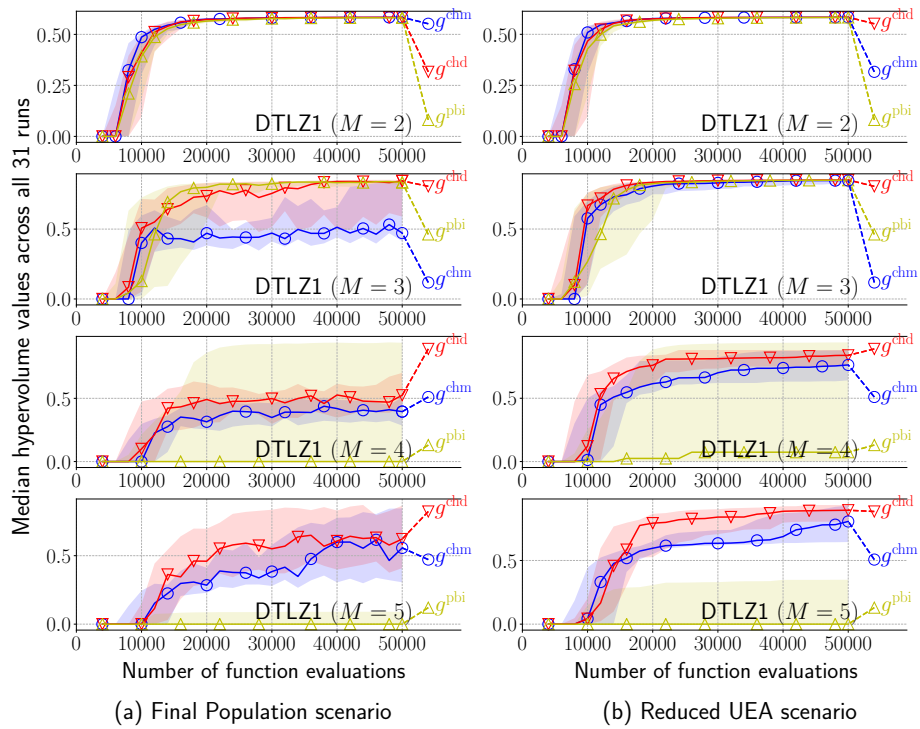


Figure S.14: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the DTLZ1 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

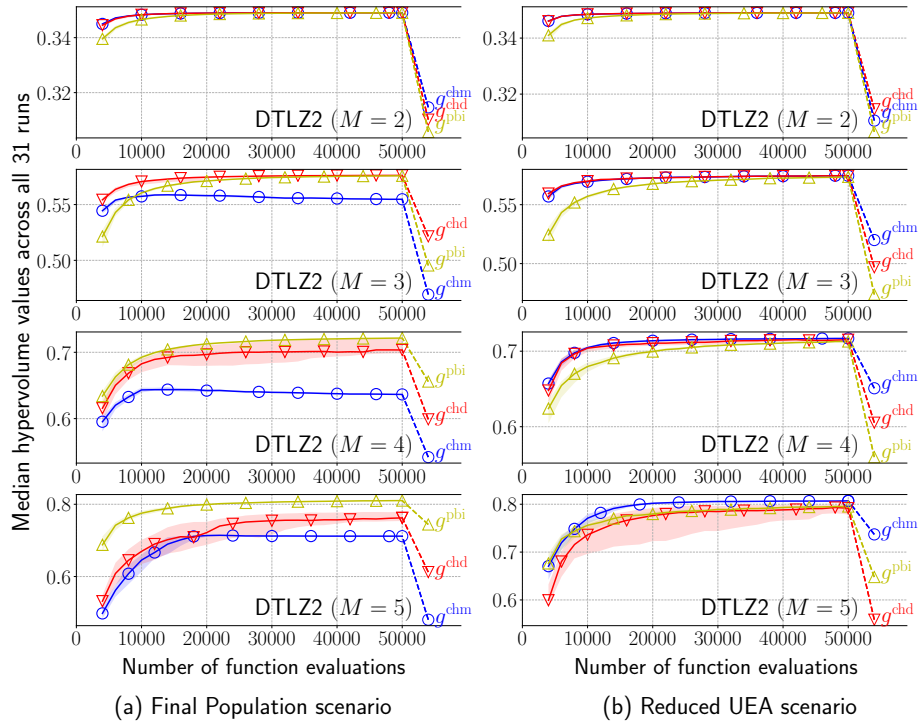


Figure S.15: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the DTLZ2 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

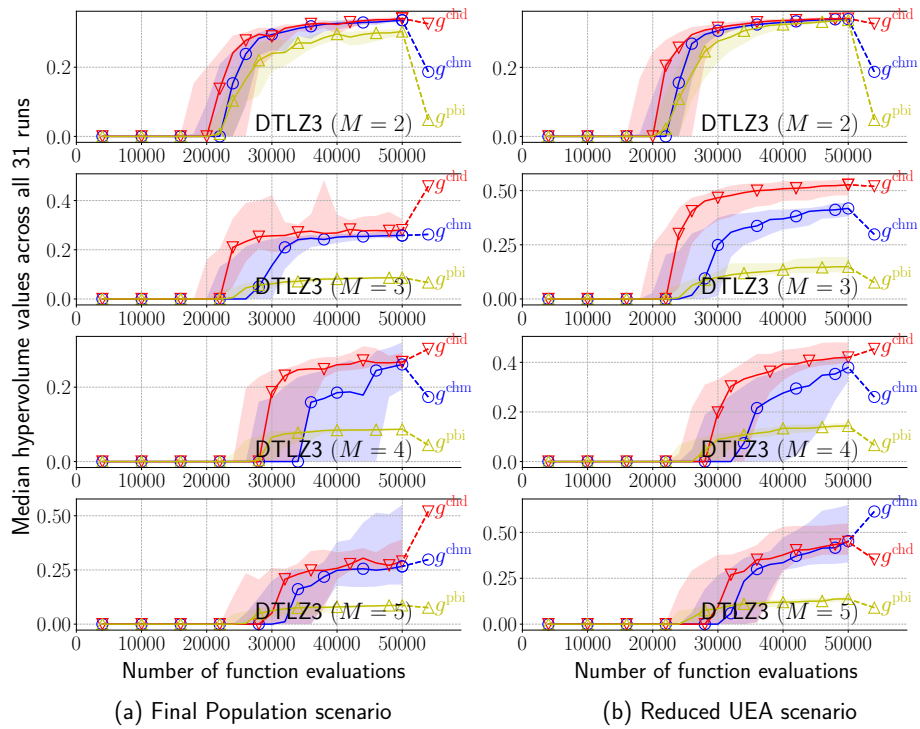


Figure S.16: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the DTLZ3 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

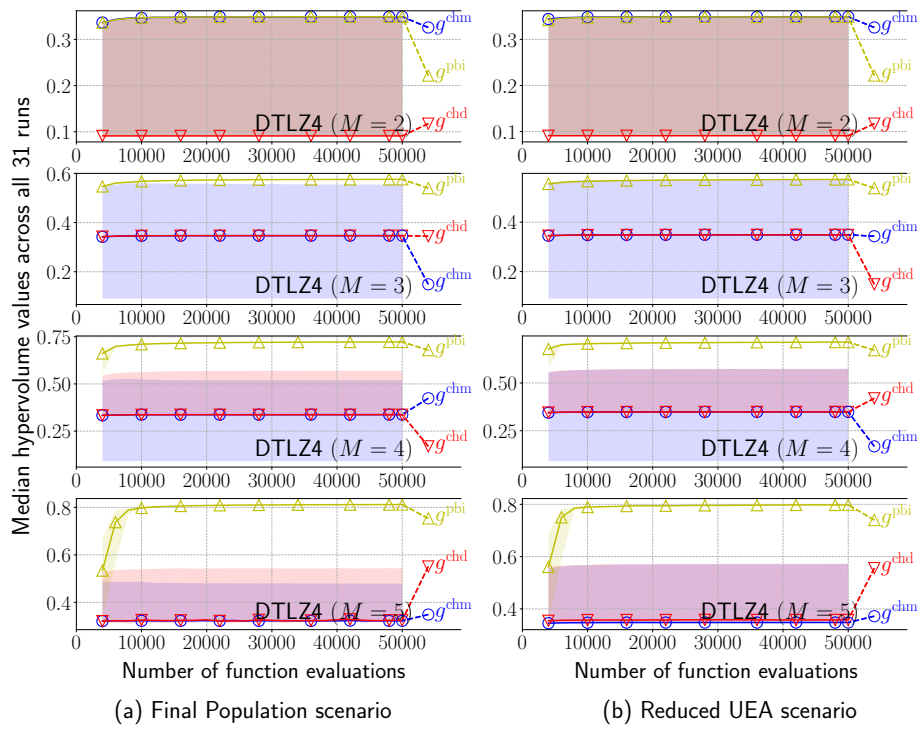


Figure S.17: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the DTLZ4 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

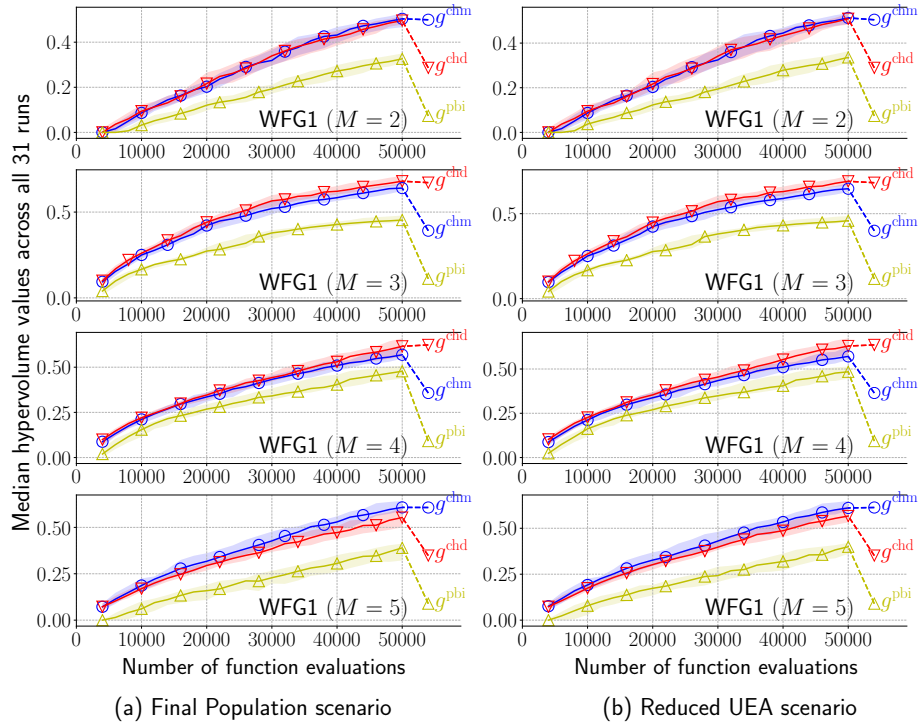


Figure S.18: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG1 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

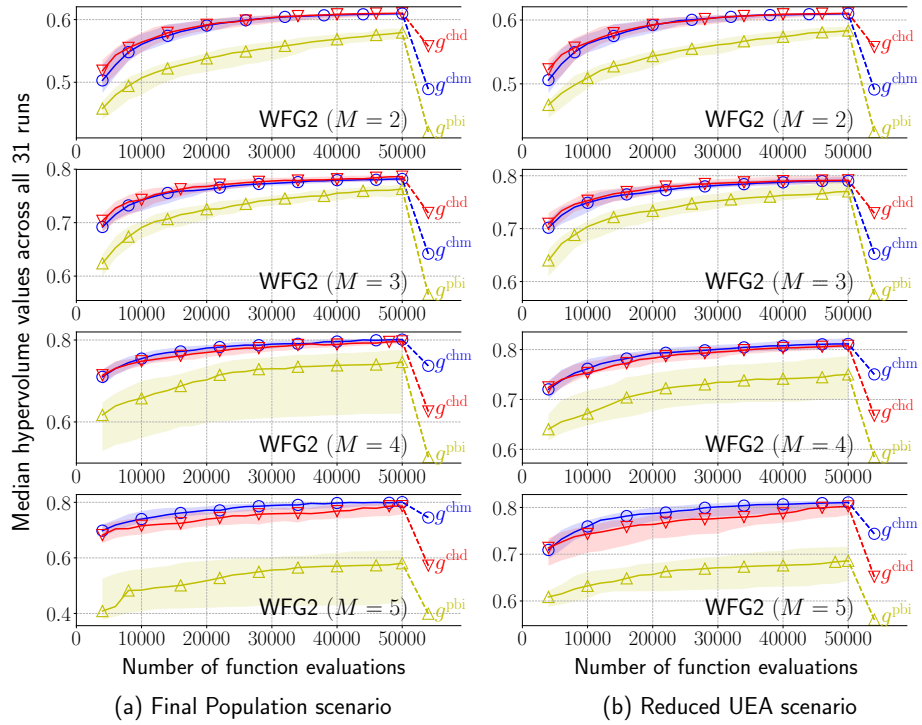


Figure S.19: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG2 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

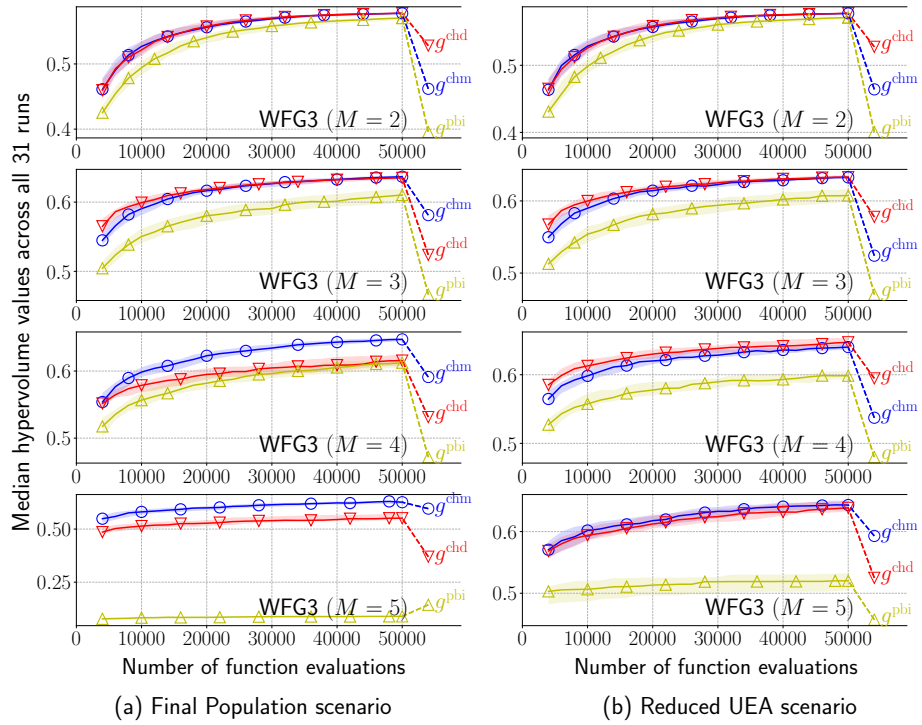


Figure S.20: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG3 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

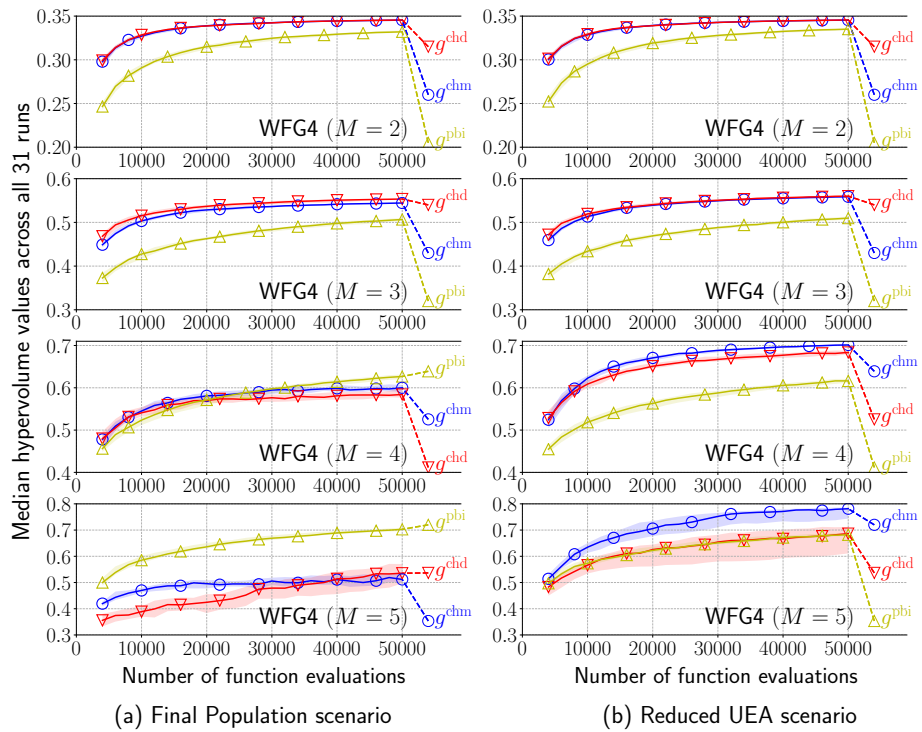


Figure S.21: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG4 problem with $M \in \{2, 3, 4, 5\}$. The horizontal (and vertical) axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

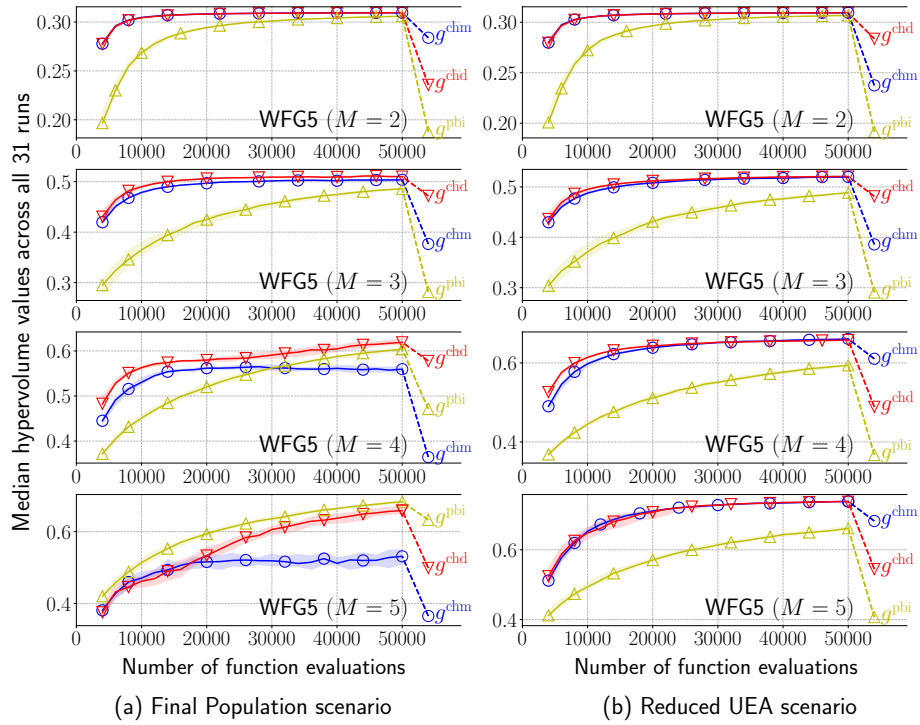


Figure S.22: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG5 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

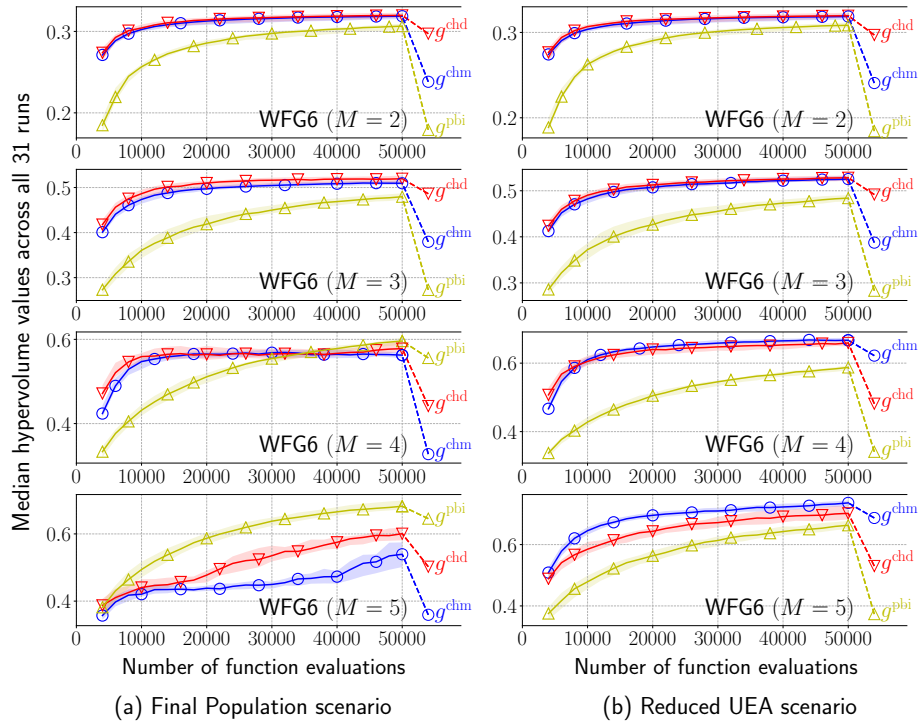


Figure S.23: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG6 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

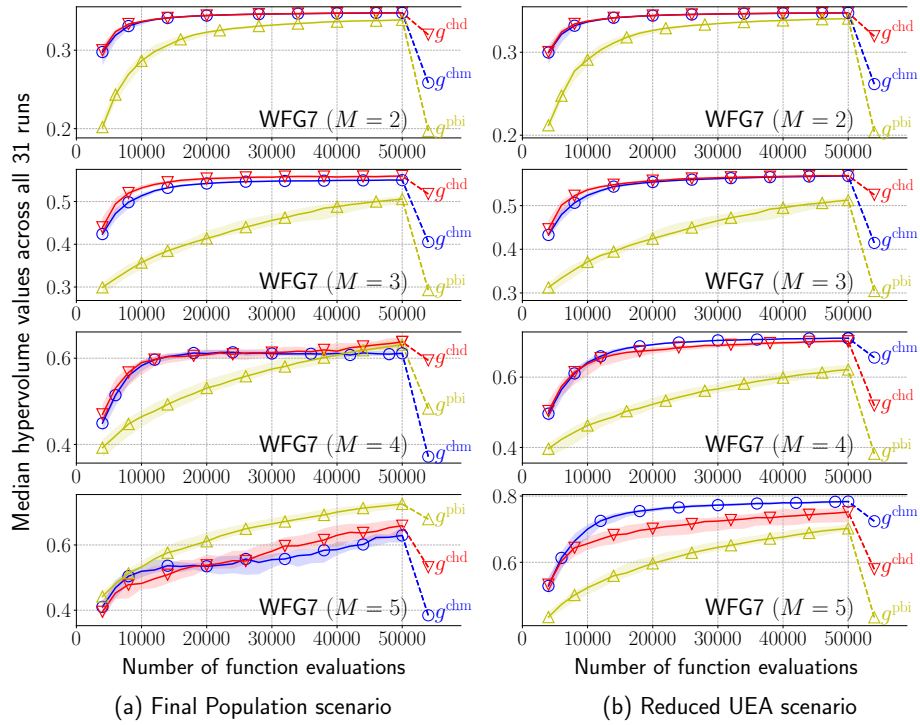


Figure S.24: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG7 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

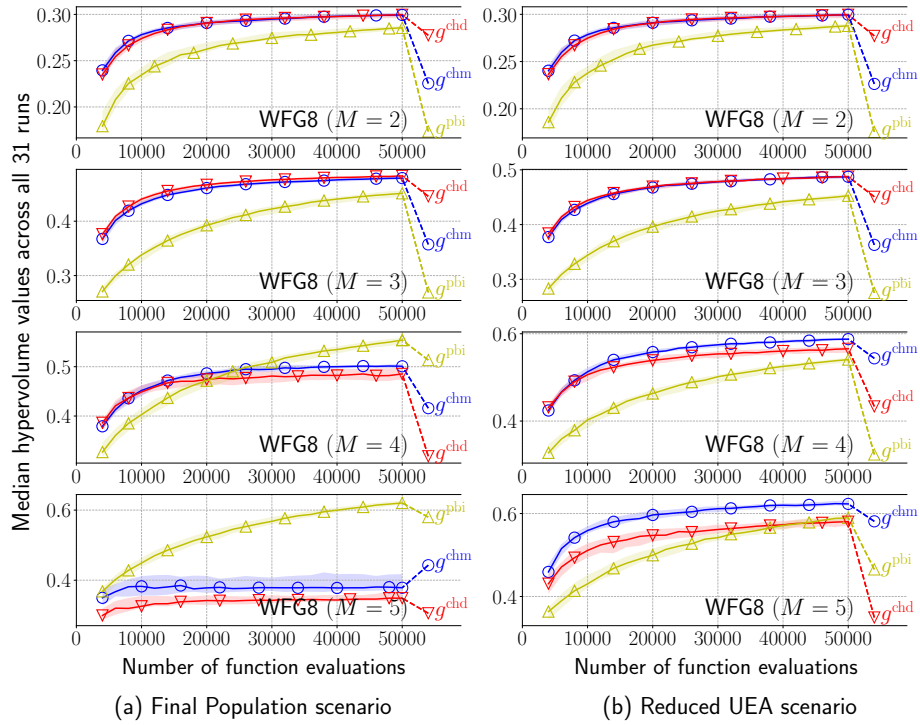


Figure S.25: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG8 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

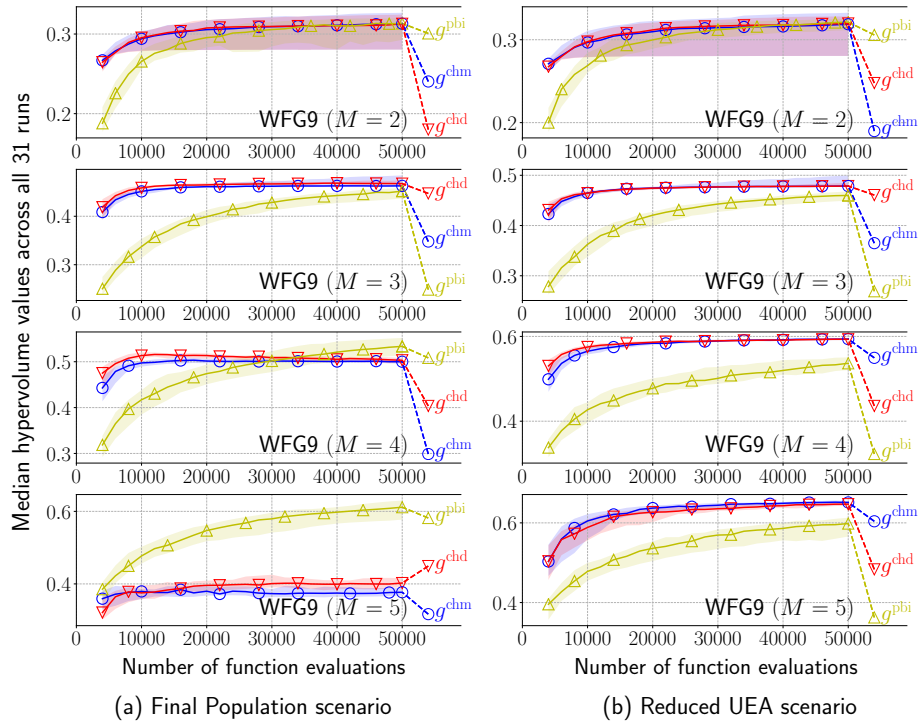


Figure S.26: Performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi} with $\theta = 5$) on the WFG9 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

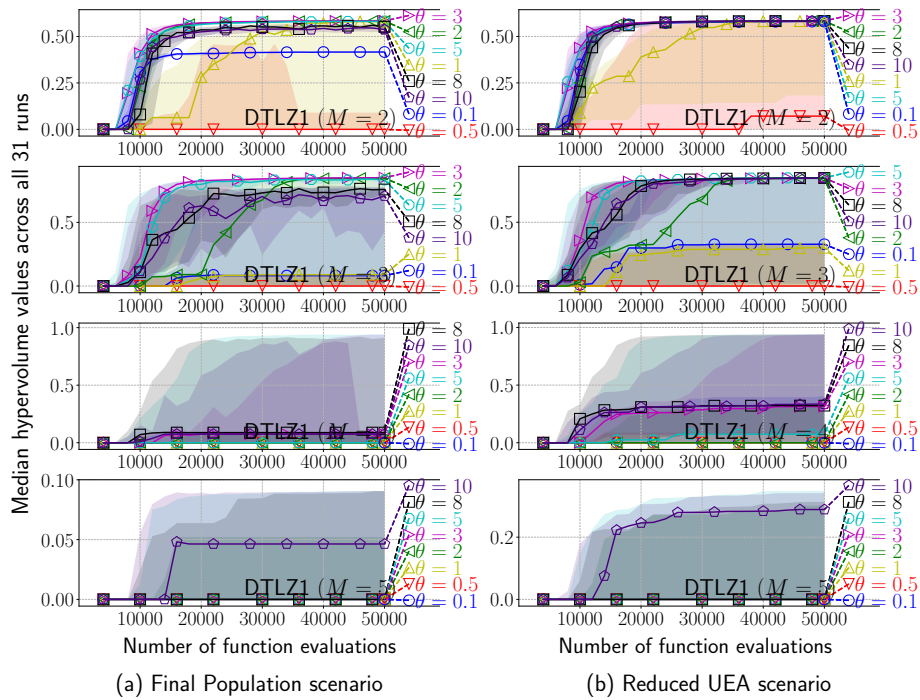


Figure S.27: Performance of MOEA/D using the PBI function g^{Pbi} with various θ values on the DTLZ1 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

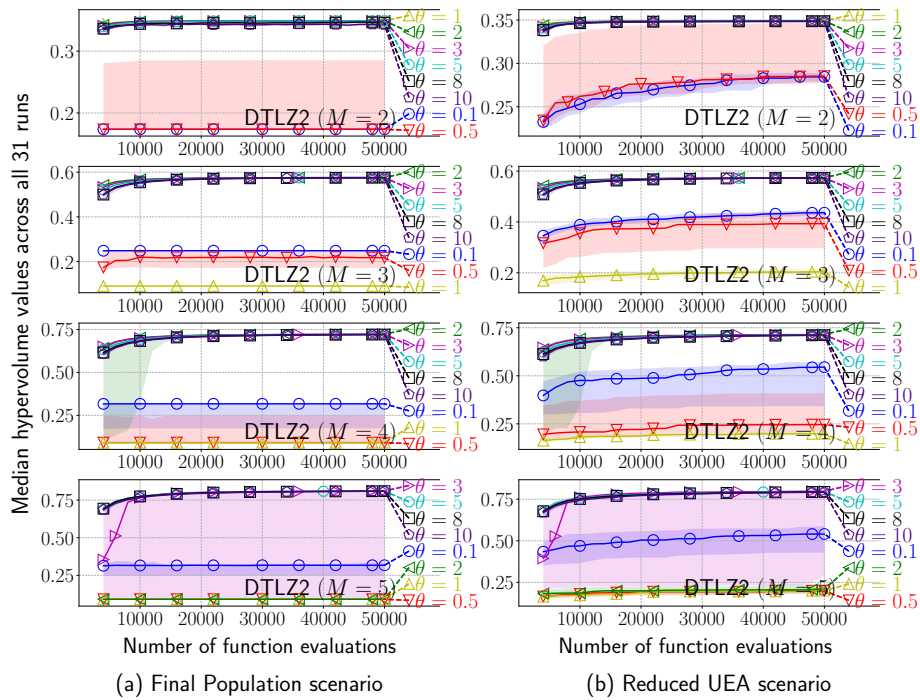


Figure S.28: Performance of MOEA/D using the PBI function g^{Pbi} with various θ values on the DTLZ2 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

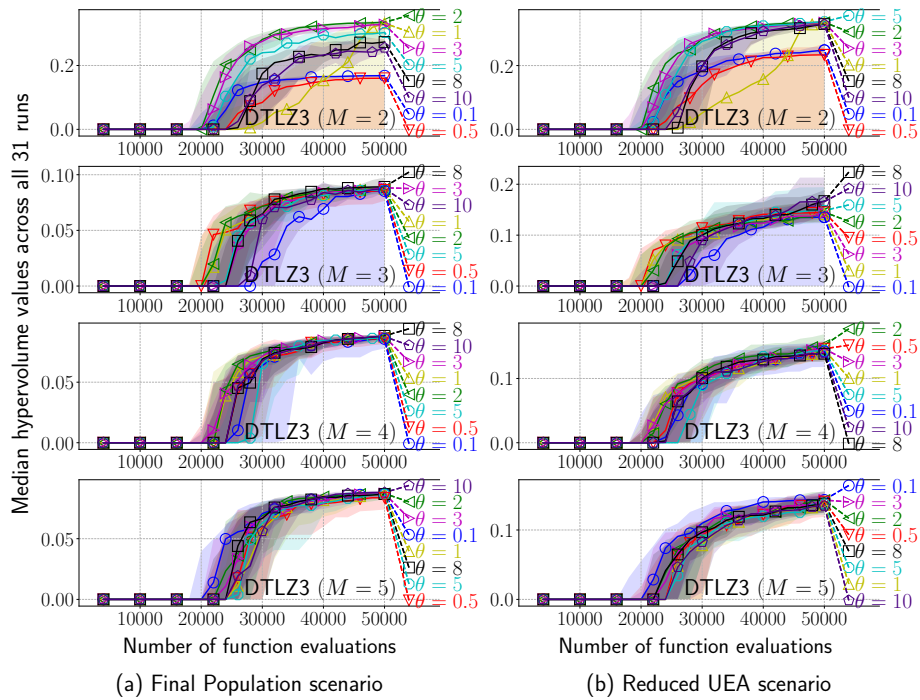


Figure S.29: Performance of MOEA/D using the PBI function g^{Pbi} with various θ values on the DTLZ3 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

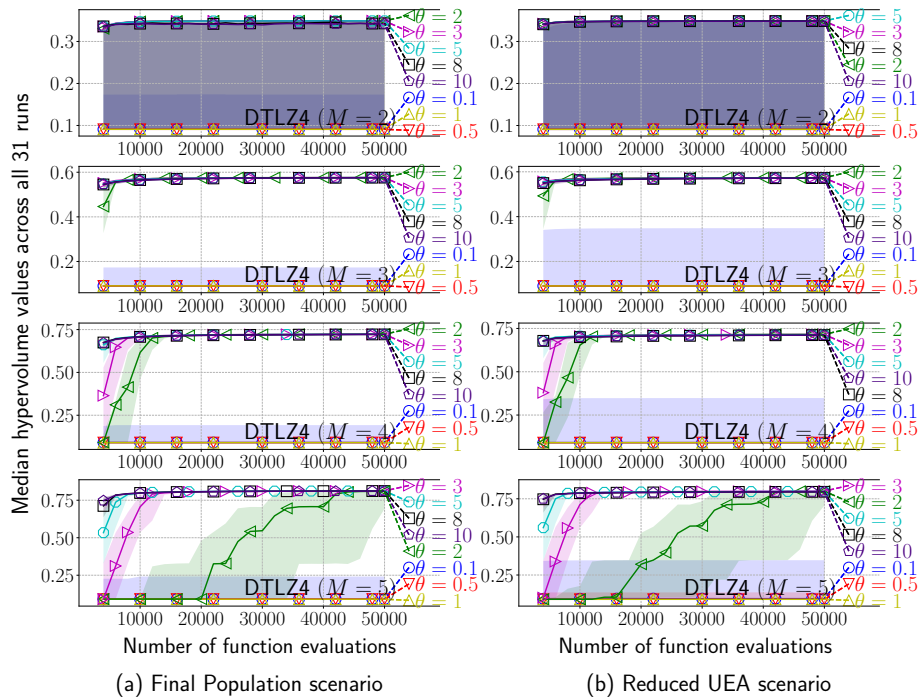


Figure S.30: Performance of MOEA/D using the PBI function g^{PBI} with various θ values on the DTLZ4 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

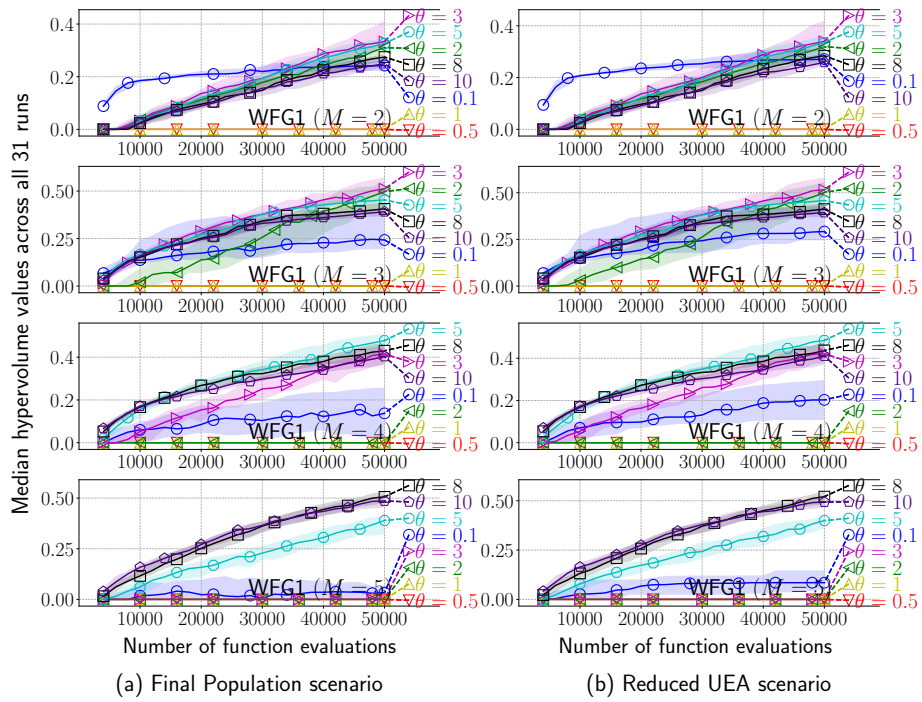


Figure S.31: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG1 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

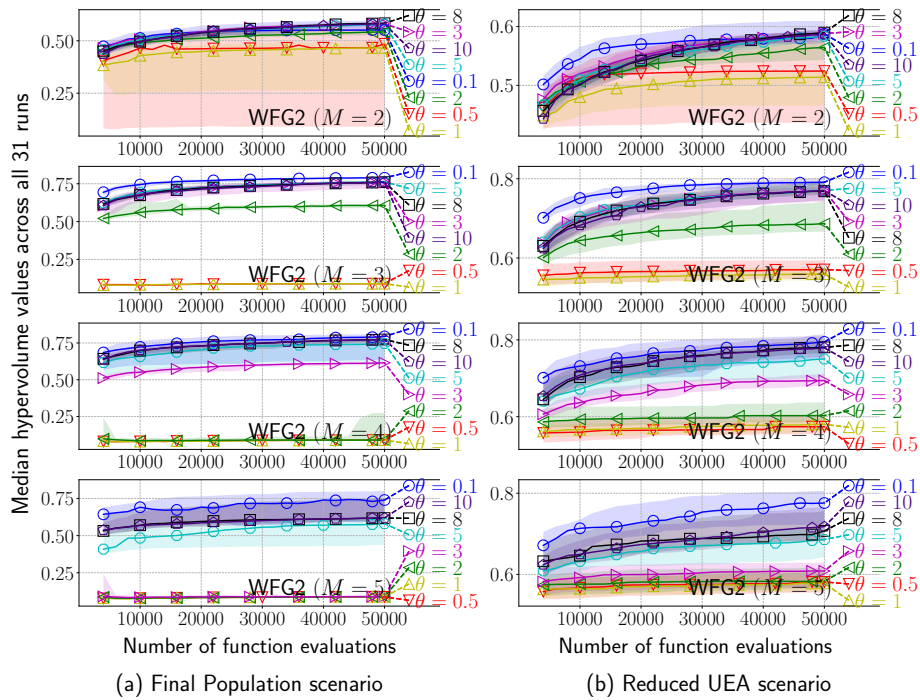


Figure S.32: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG2 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

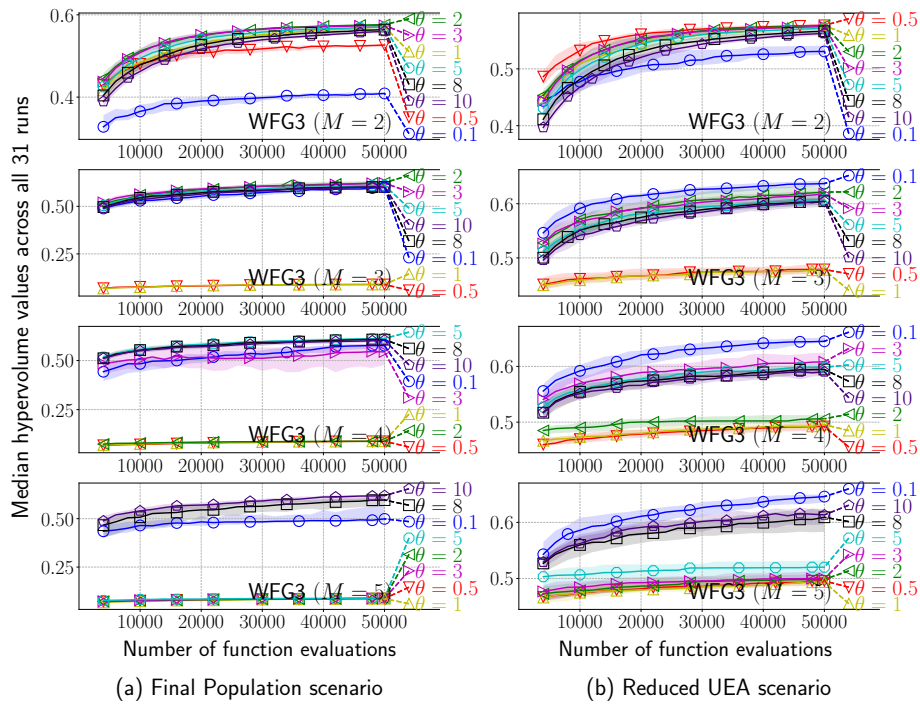


Figure S.33: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG3 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

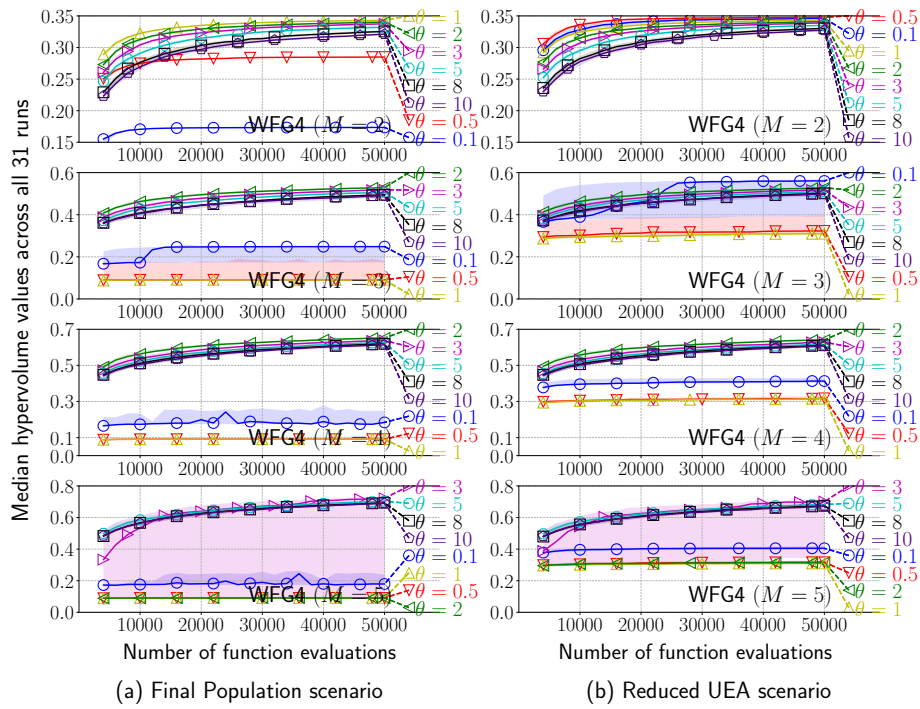


Figure S.34: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG4 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

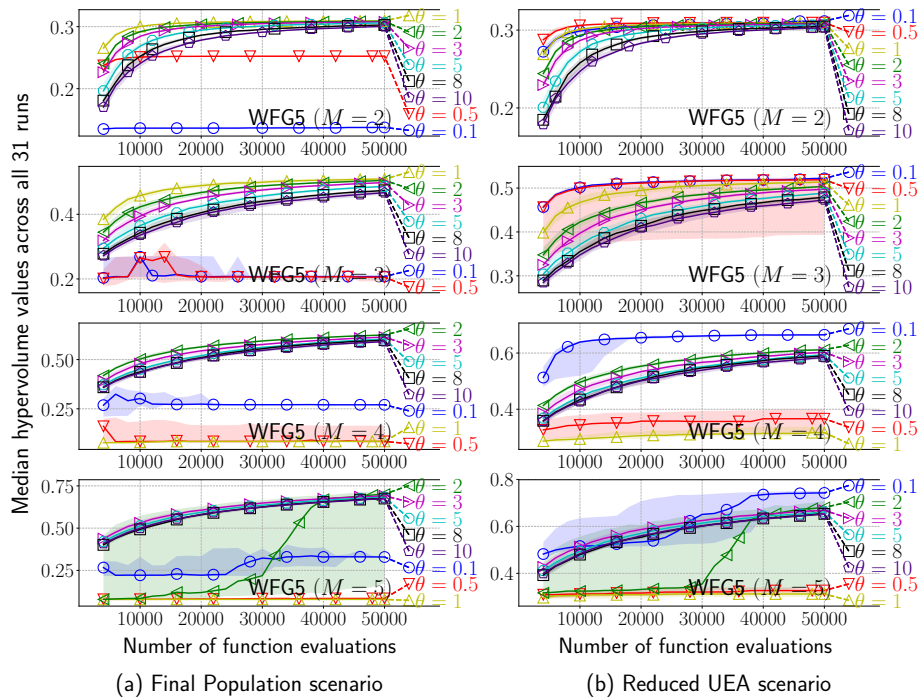


Figure S.35: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG5 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

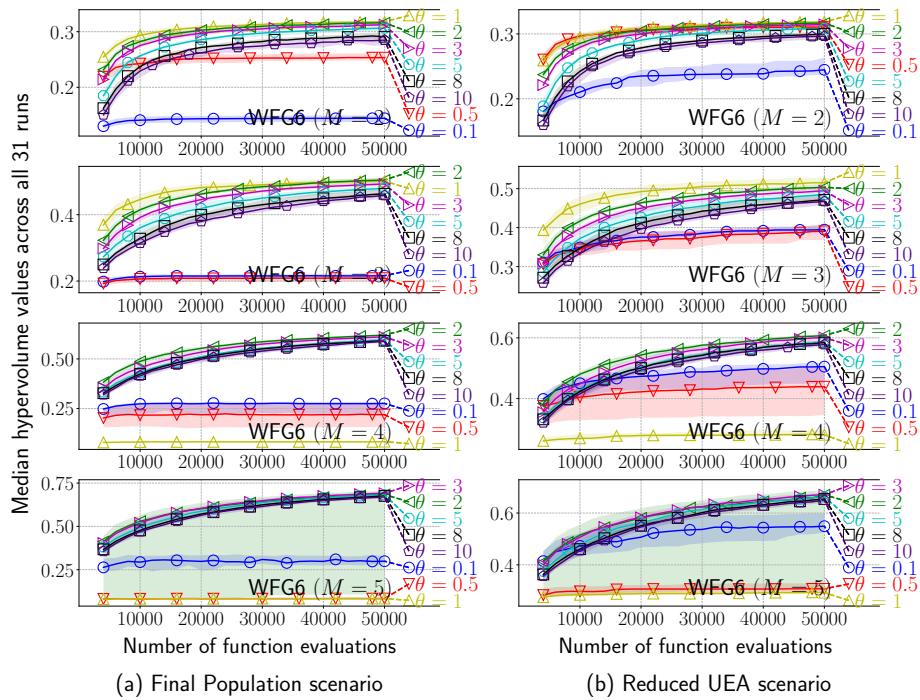


Figure S.36: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG6 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

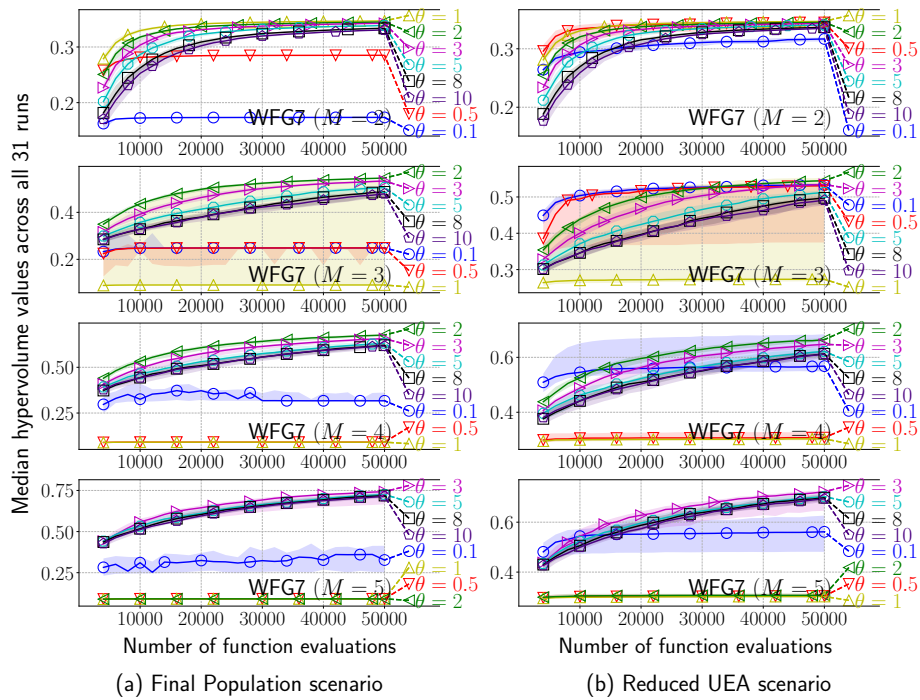


Figure S.37: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG7 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

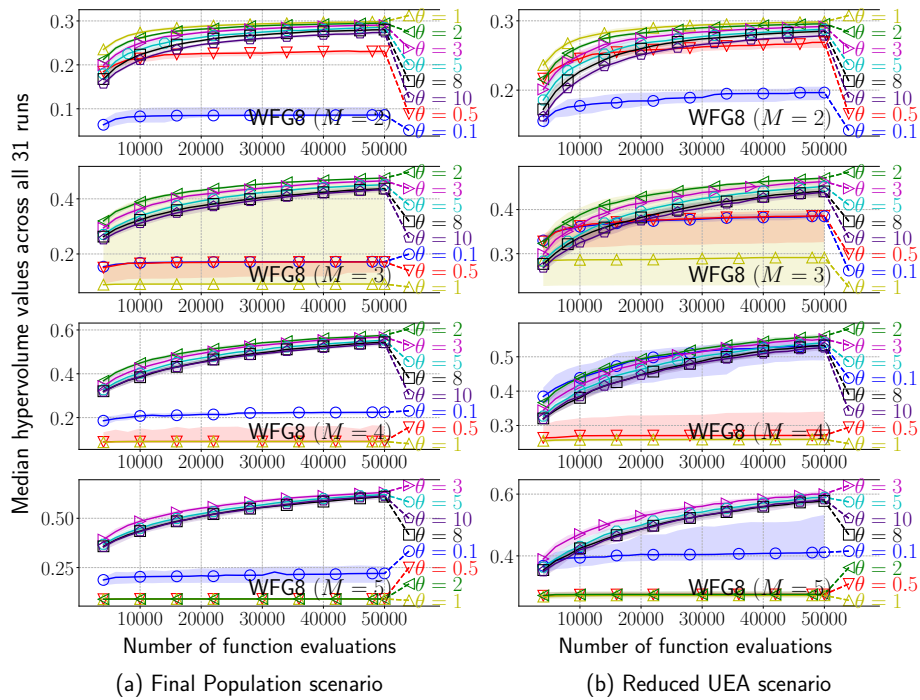


Figure S.38: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG8 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

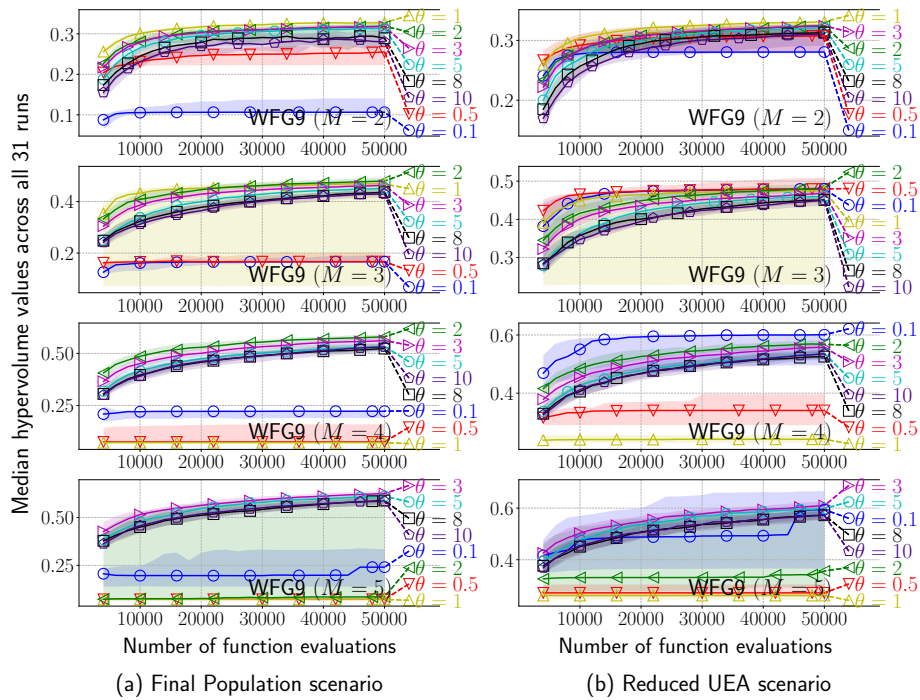


Figure S.39: Performance of MOEA/D using the PBI function g^{pbi} with various θ values on the WFG9 problem with $M \in \{2, 3, 4, 5\}$. The horizontal and vertical axes represent the number of function evaluations and the HV values, respectively. The shaded area indicates 25-75 percentiles.

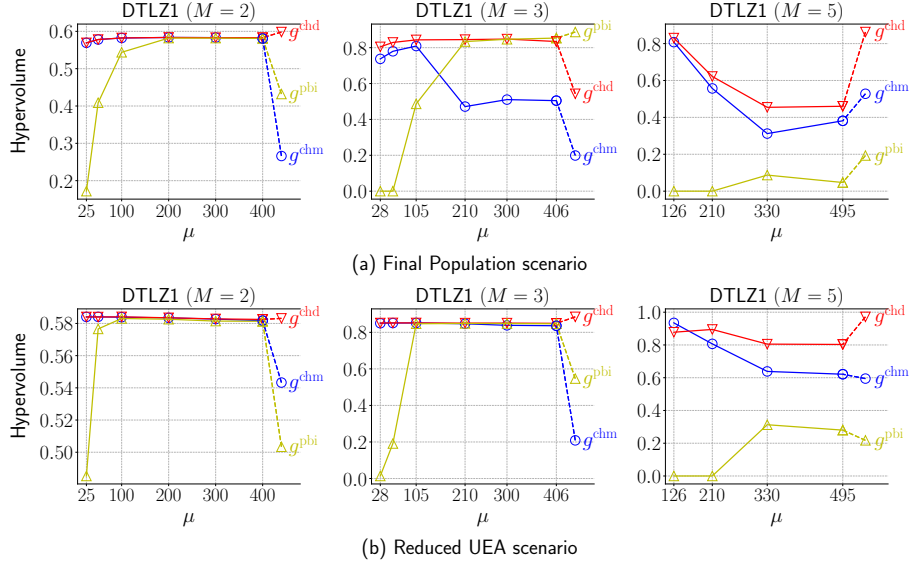


Figure S.40: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the DTLZ1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

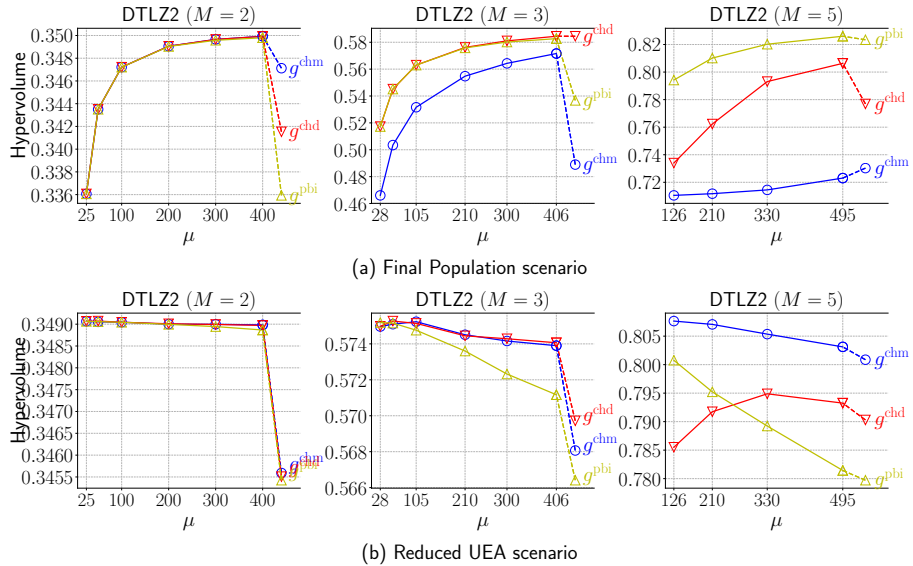


Figure S.41: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the DTLZ2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

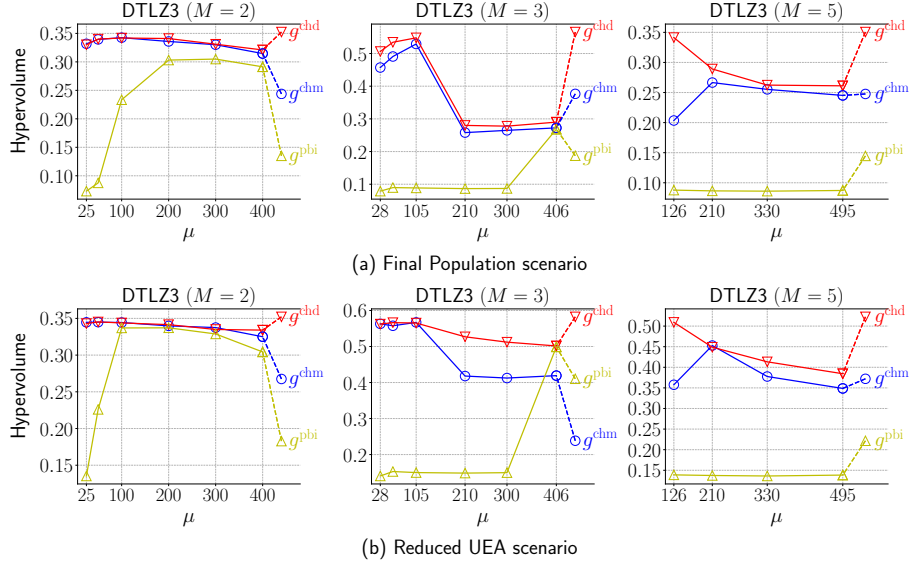


Figure S.42: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the DTLZ3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

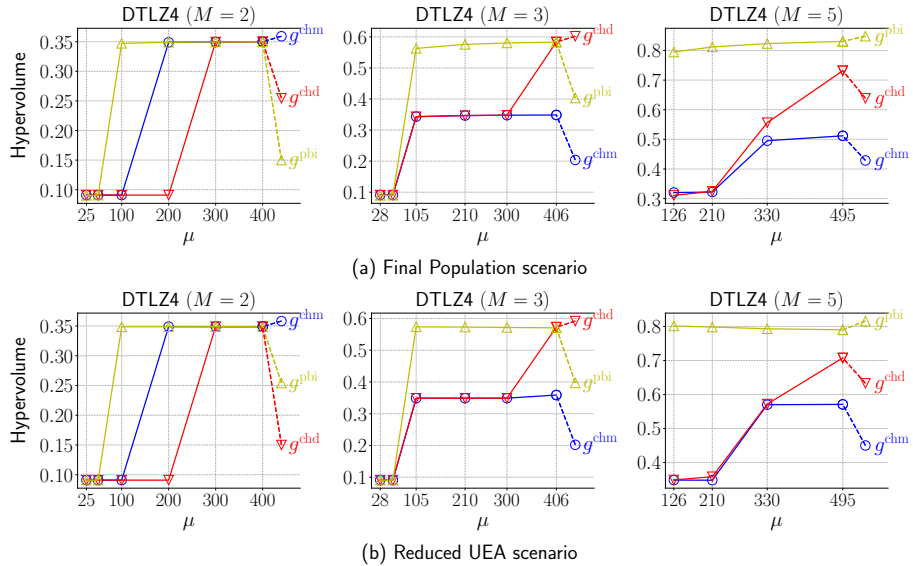


Figure S.43: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the DTLZ4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

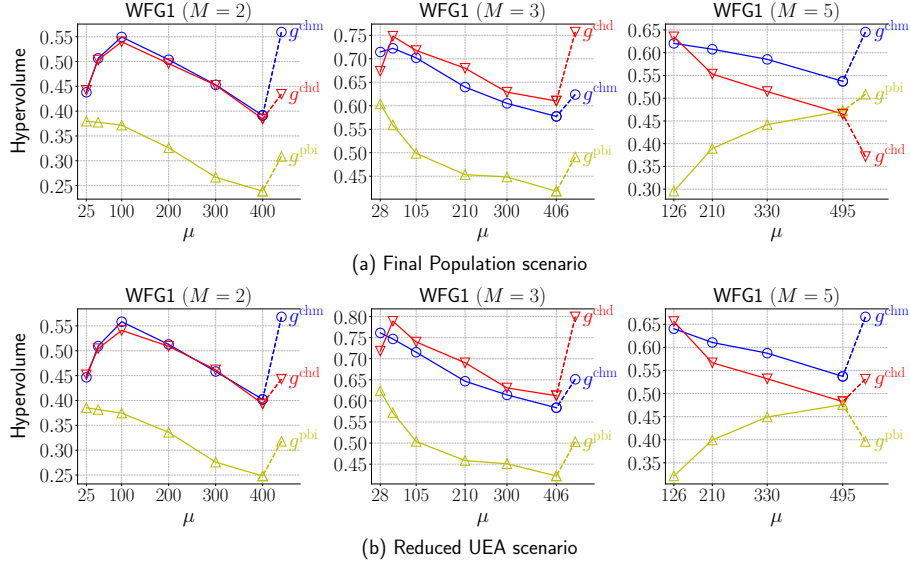


Figure S.44: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

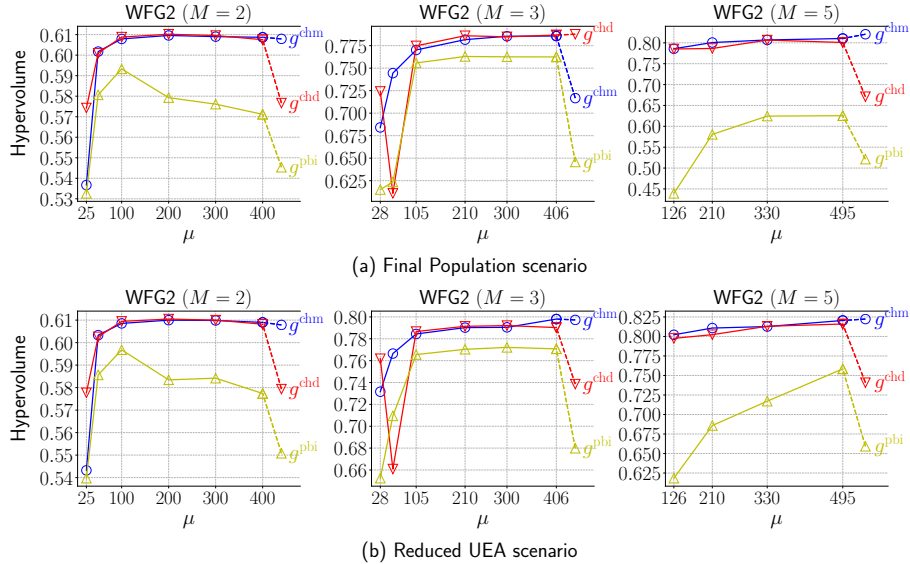


Figure S.45: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

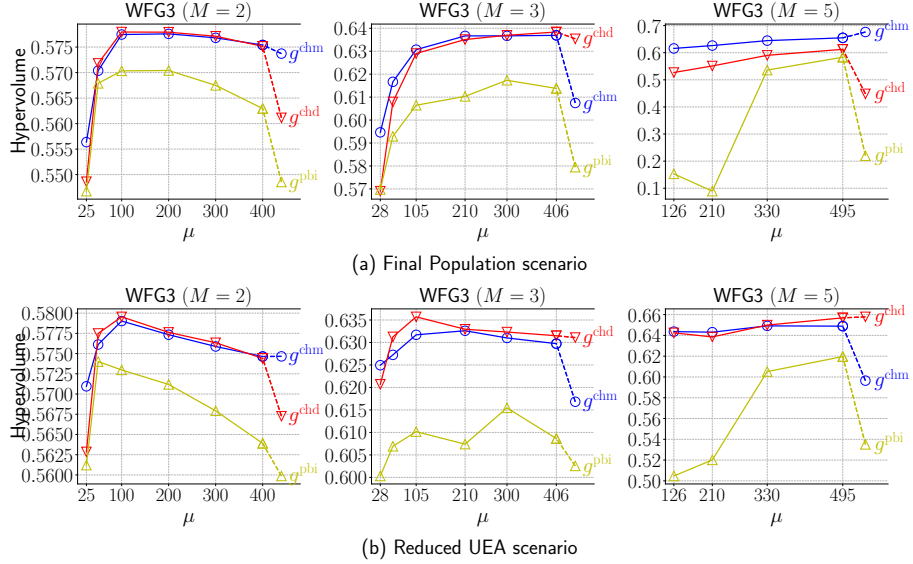


Figure S.46: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

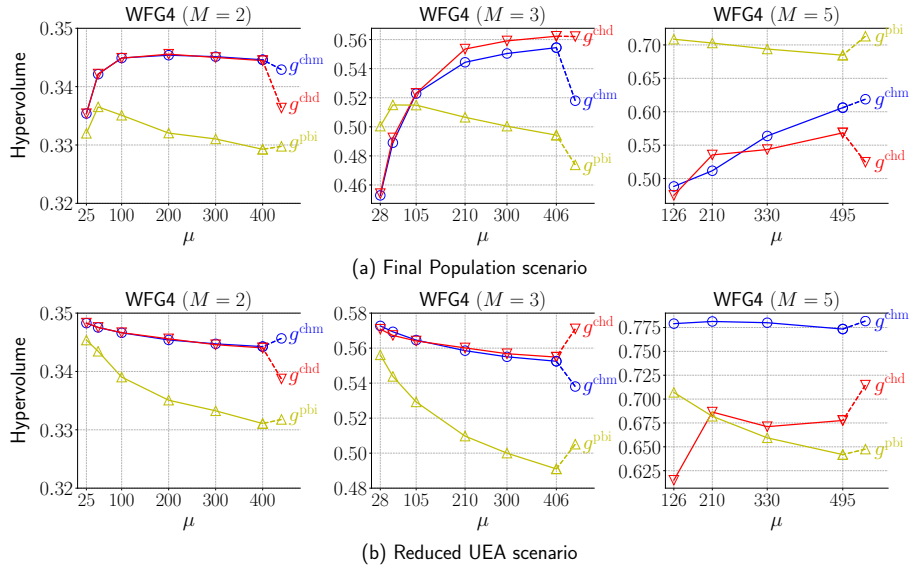


Figure S.47: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

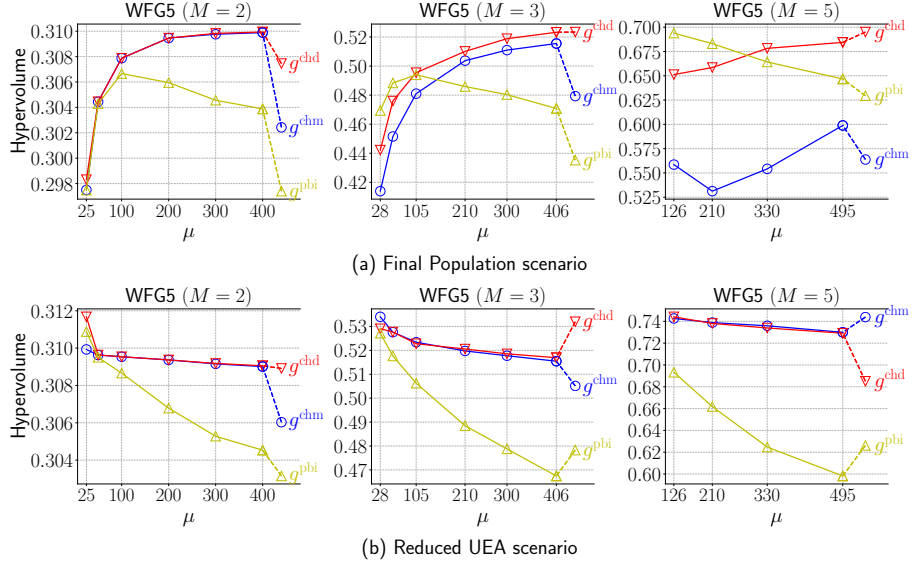


Figure S.48: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG5 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

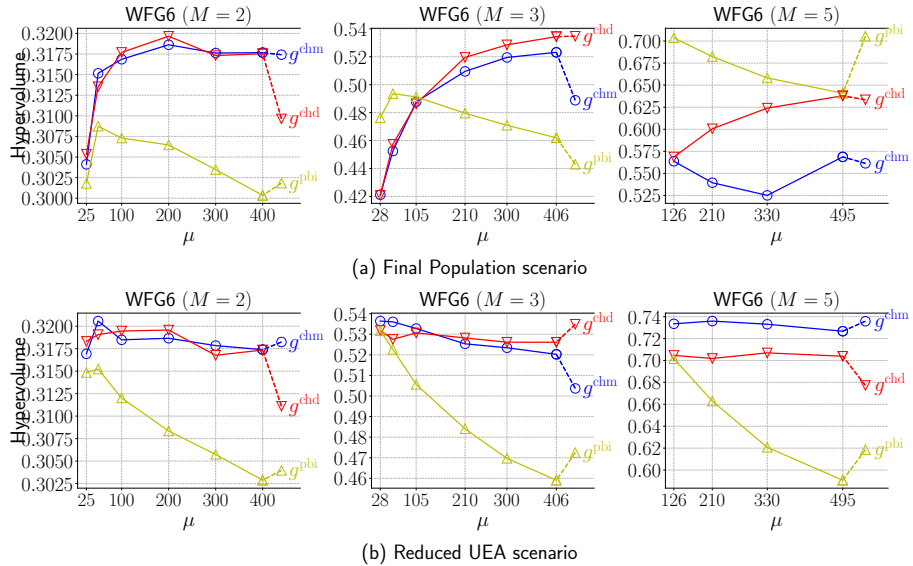


Figure S.49: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG6 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

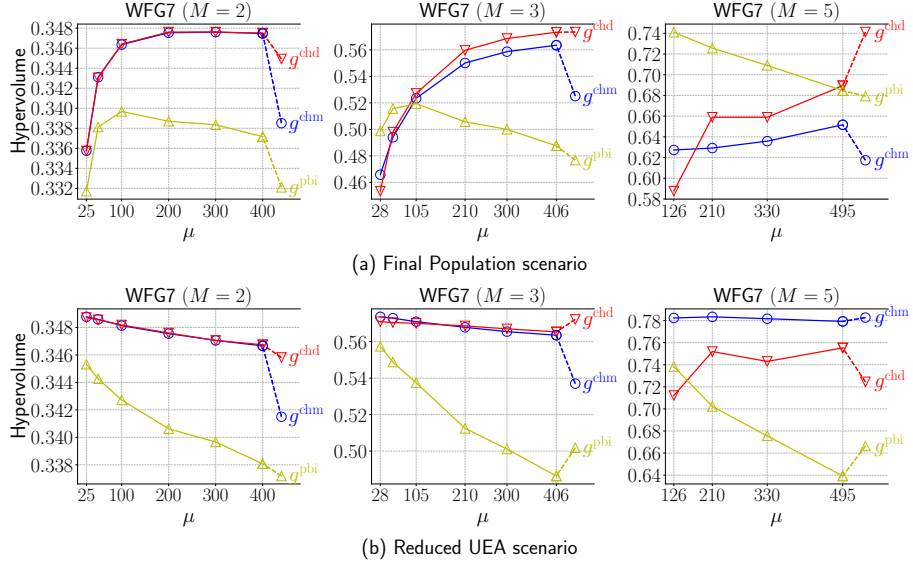


Figure S.50: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG7 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

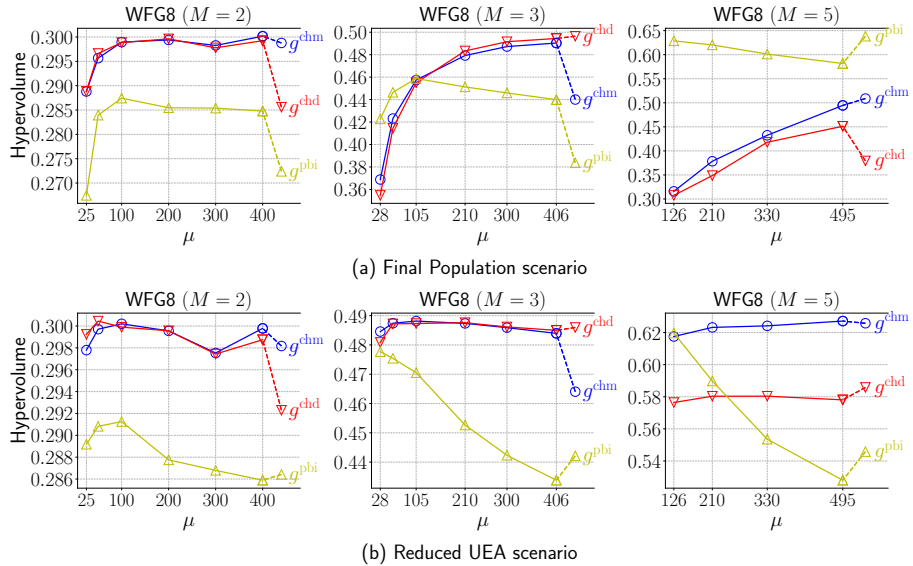


Figure S.51: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG8 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

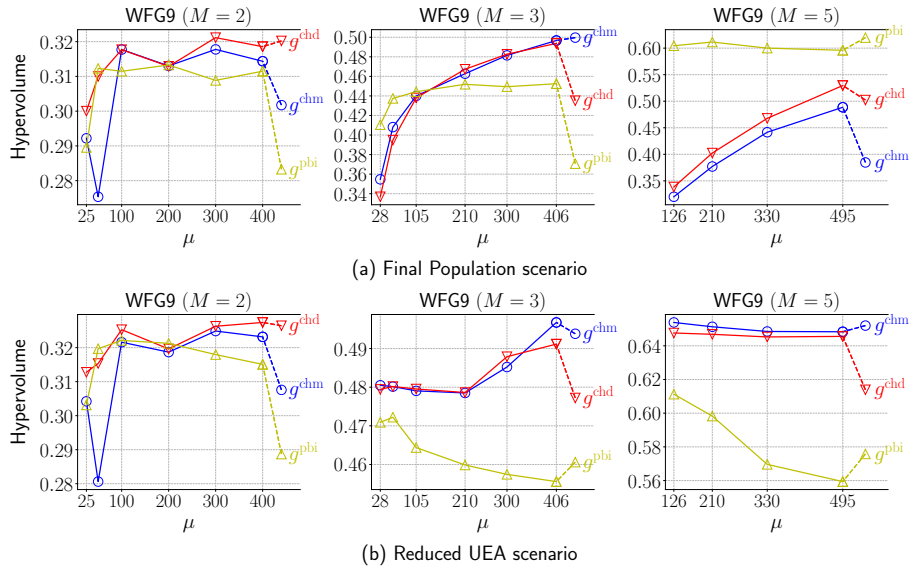


Figure S.52: Influence of μ on the performance of MOEA/D with the three scalarizing functions (g^{chm} , g^{chd} , and g^{pbi}) on the WFG9 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

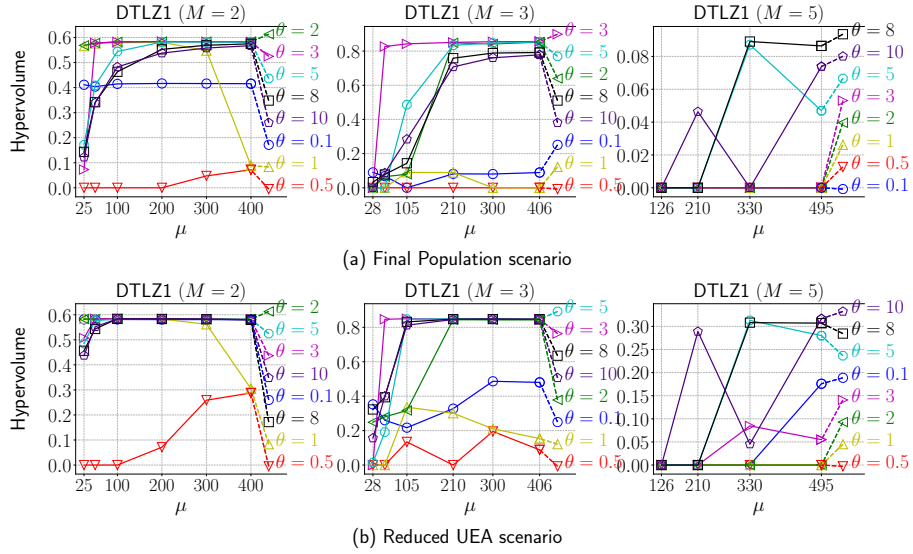


Figure S.53: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

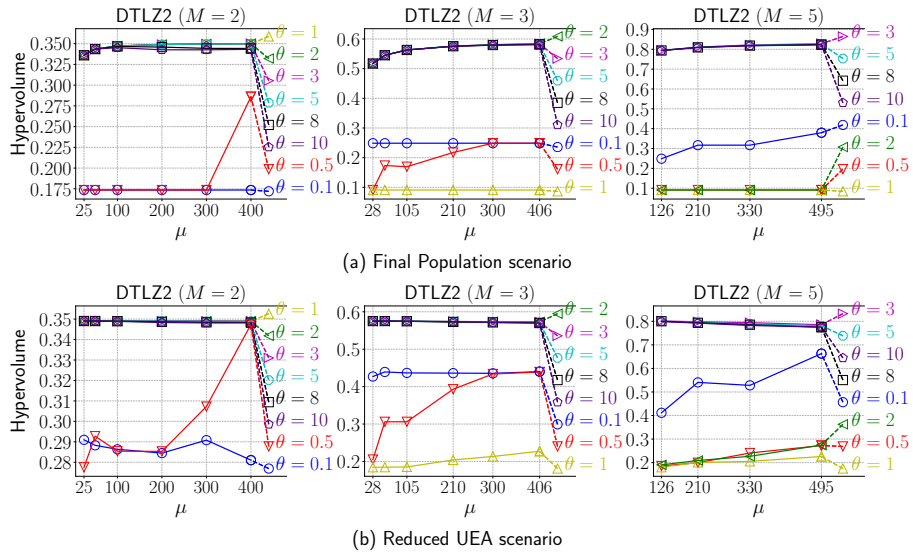


Figure S.54: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

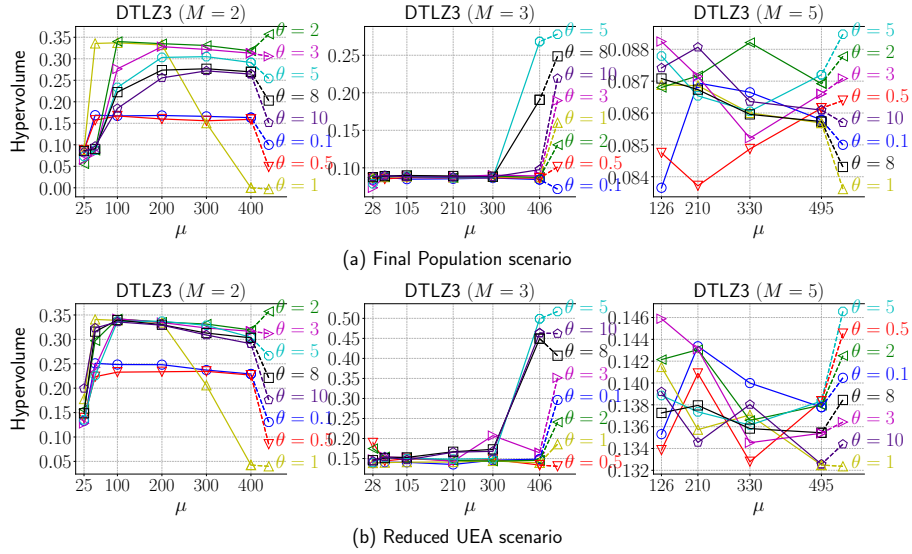


Figure S.55: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

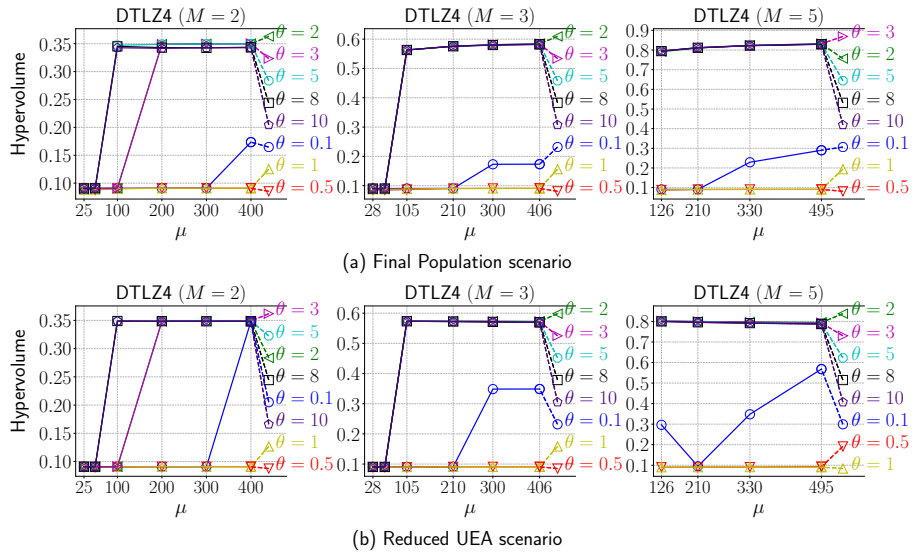


Figure S.56: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

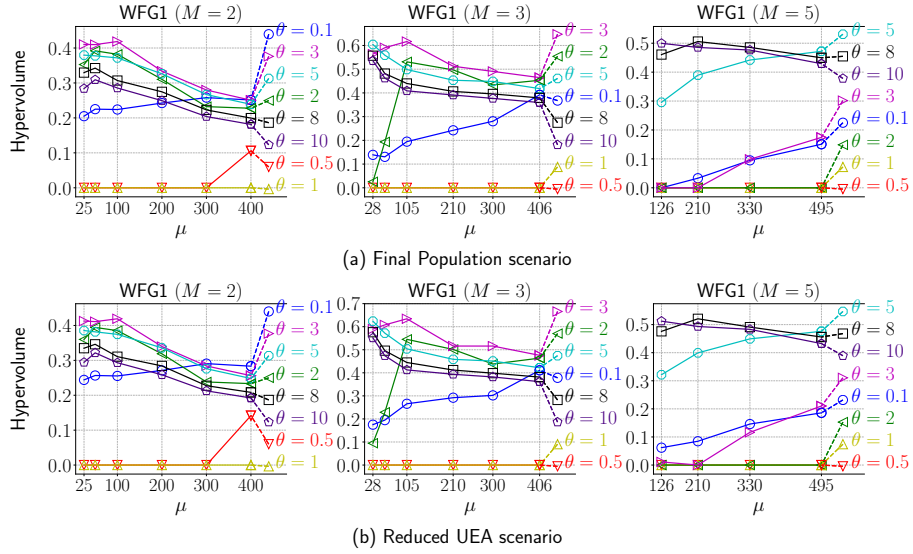


Figure S.57: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

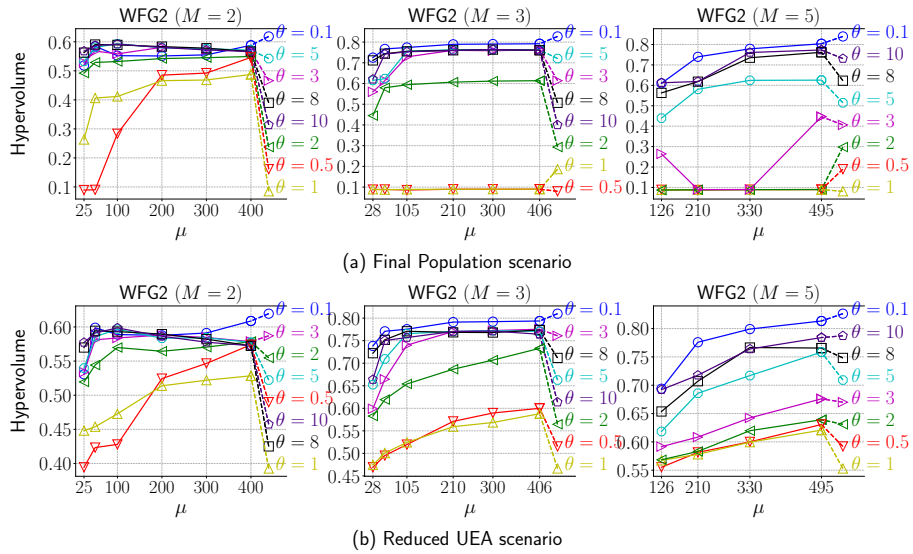


Figure S.58: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

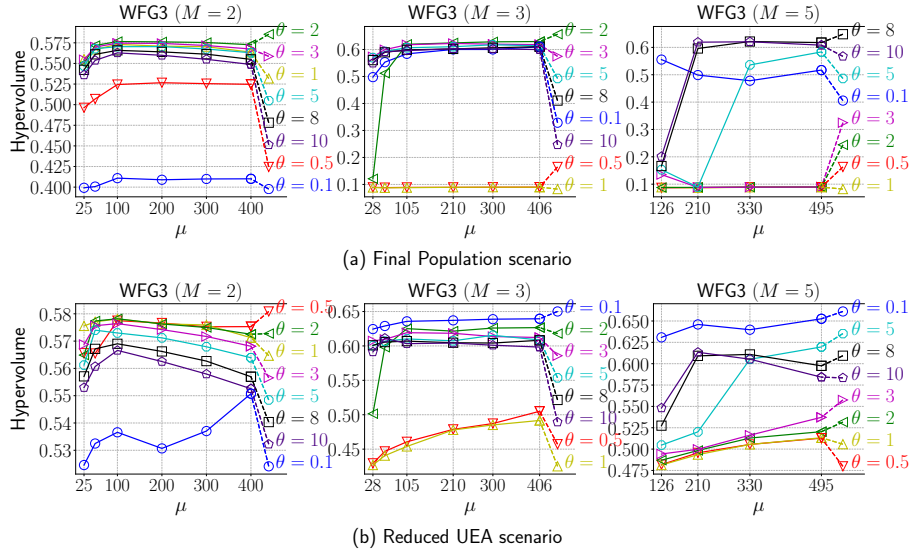


Figure S.59: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

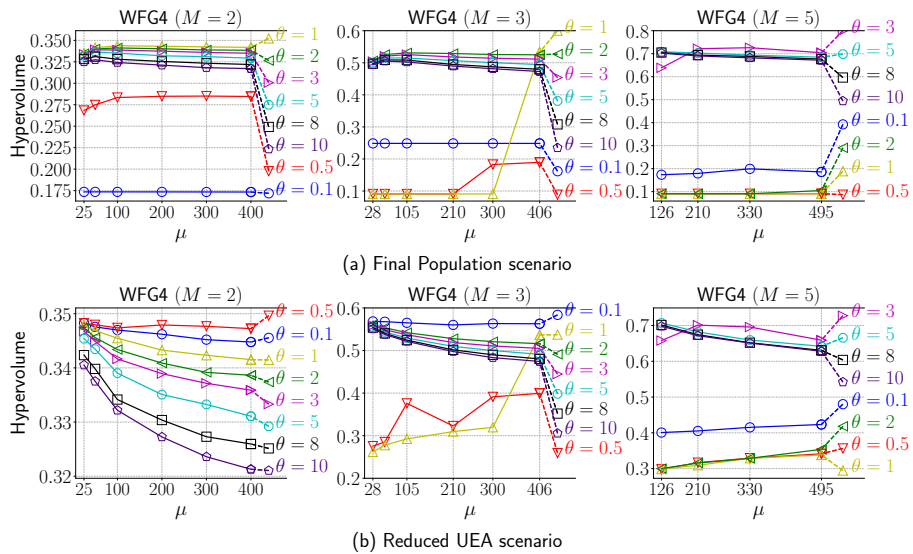


Figure S.60: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

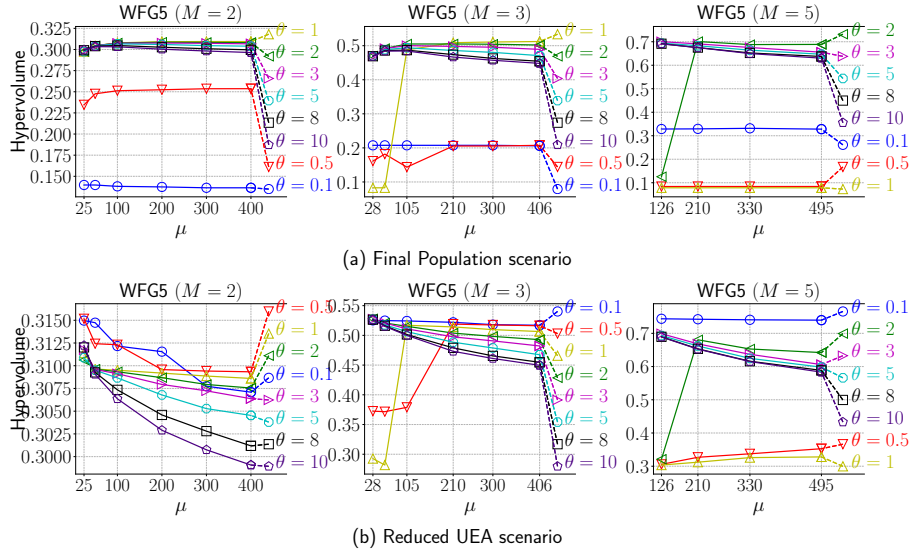


Figure S.61: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG5 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

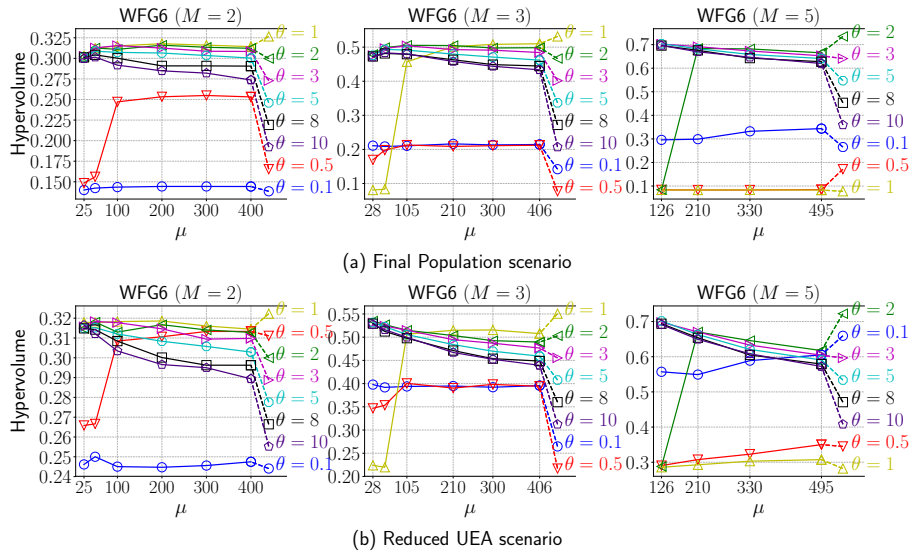


Figure S.62: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG6 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

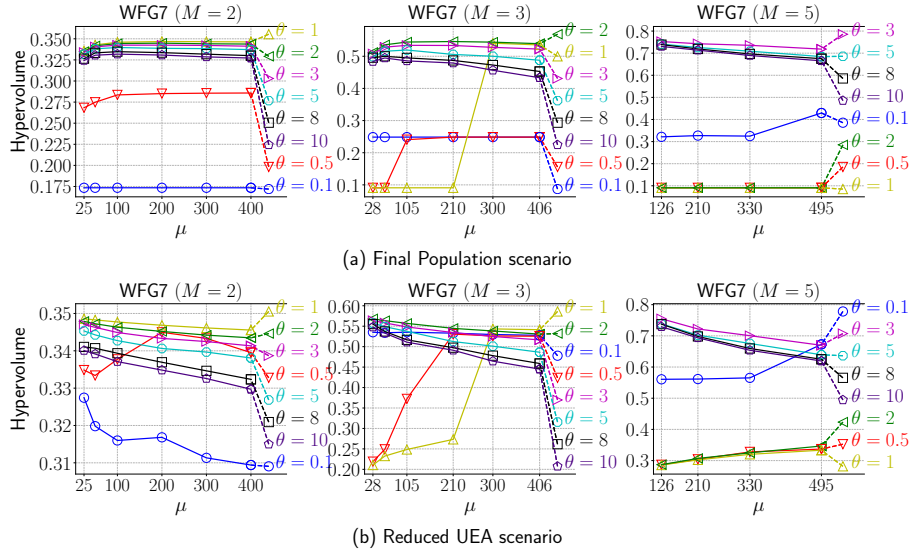


Figure S.63: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG7 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

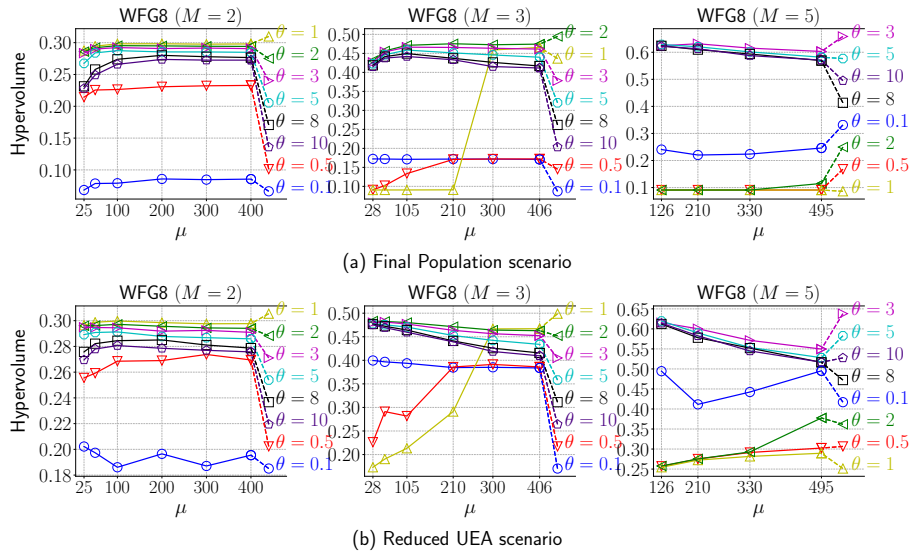


Figure S.64: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG8 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

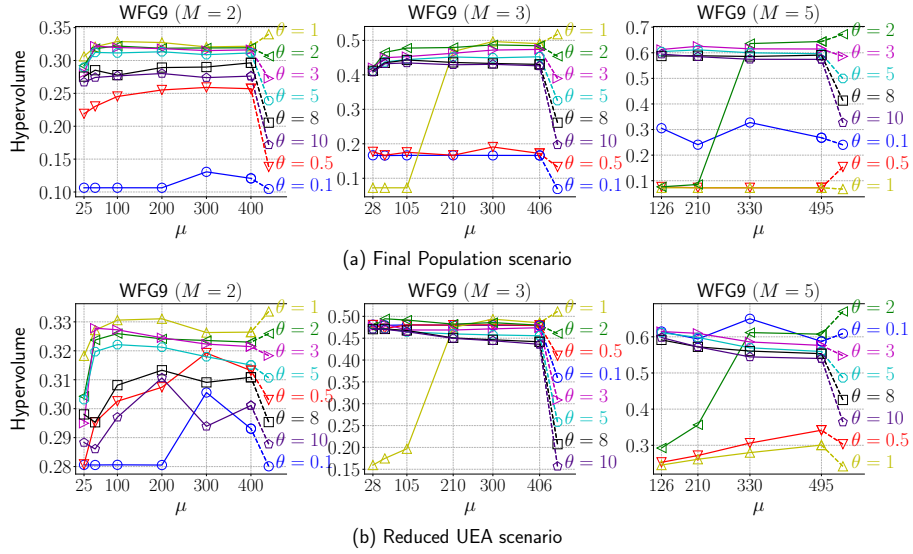


Figure S.65: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG9 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

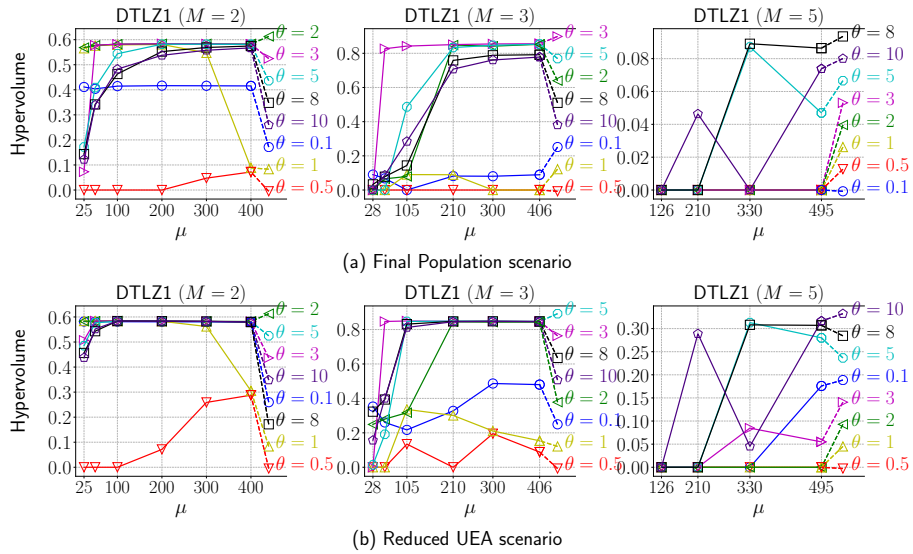


Figure S.66: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

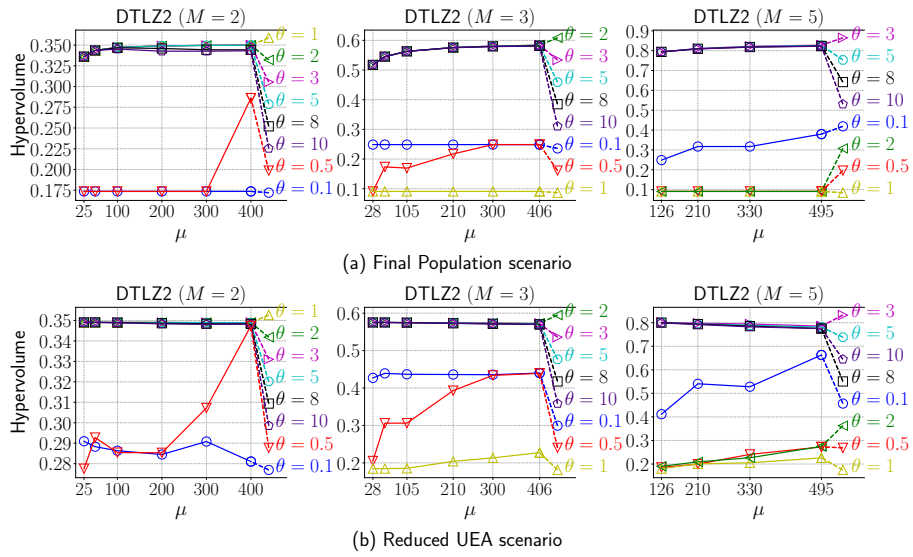


Figure S.67: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

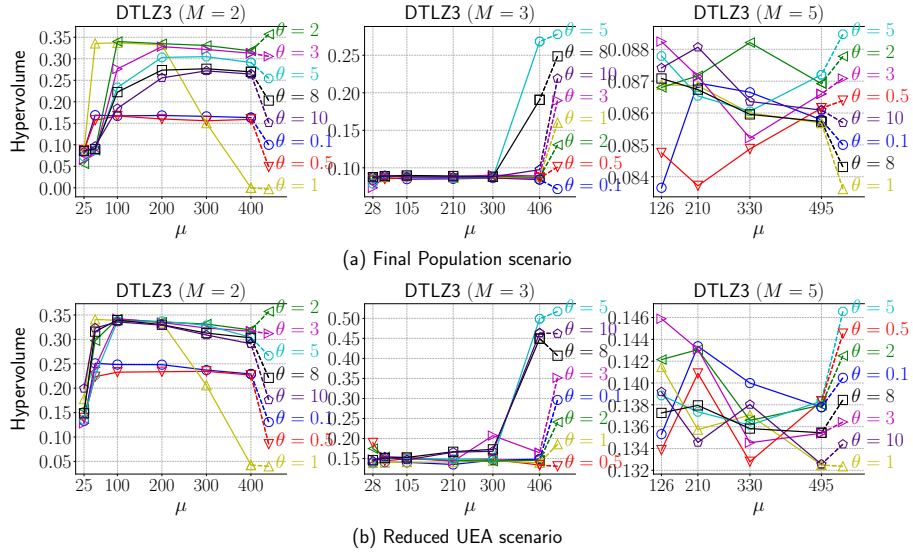


Figure S.68: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

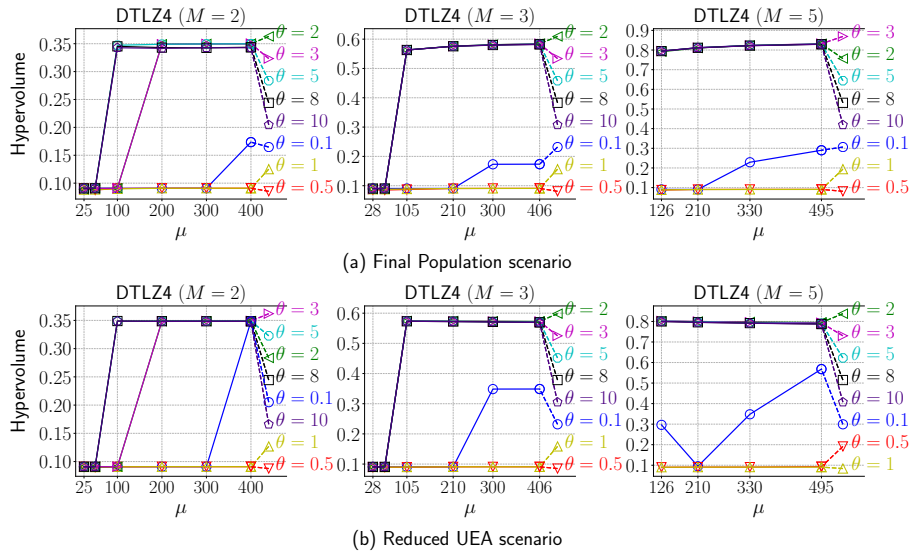


Figure S.69: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the DTLZ4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

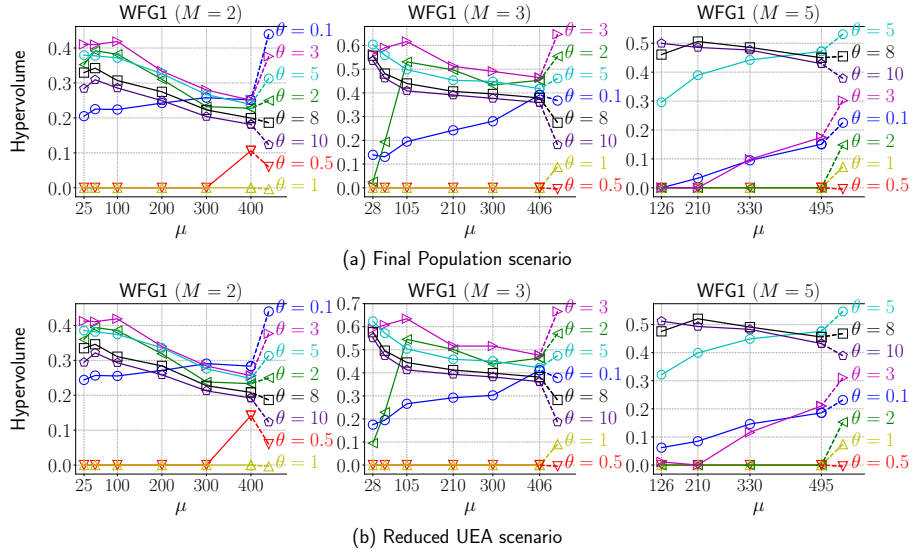


Figure S.70: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

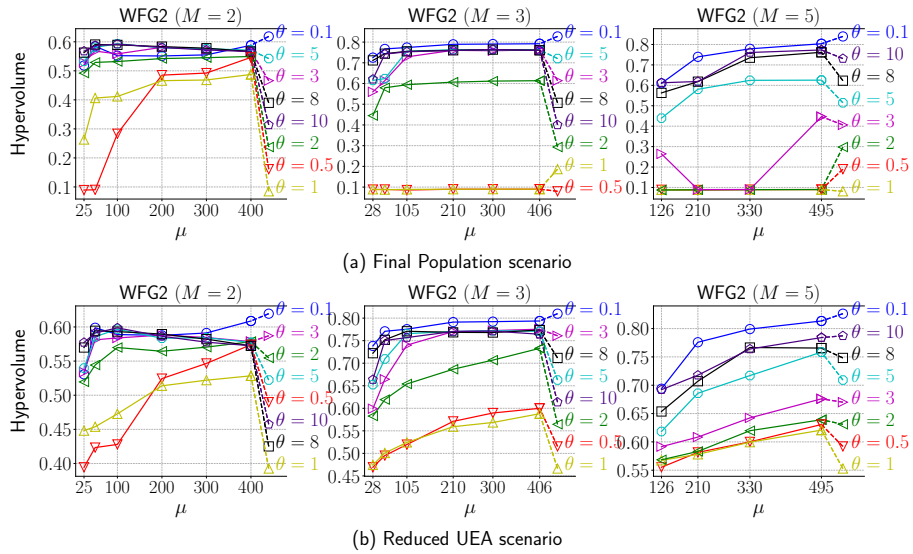


Figure S.71: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

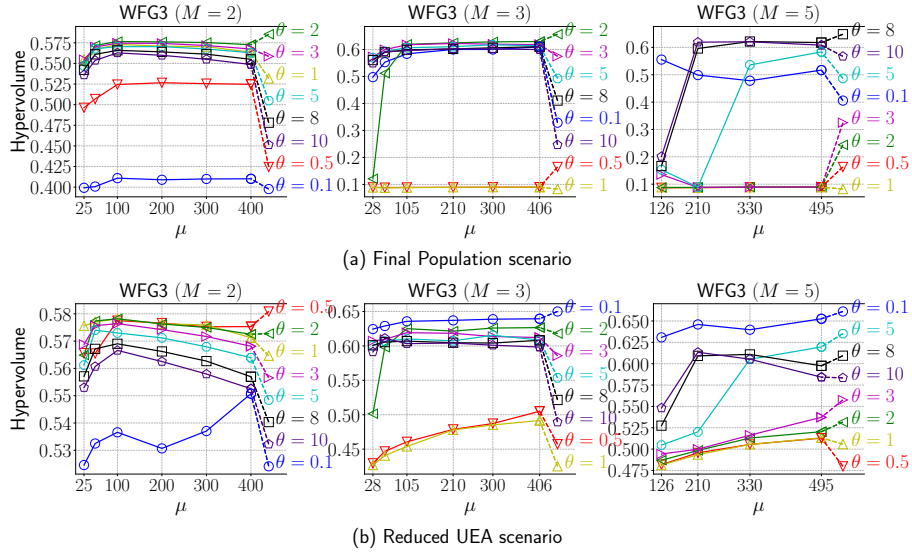


Figure S.72: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

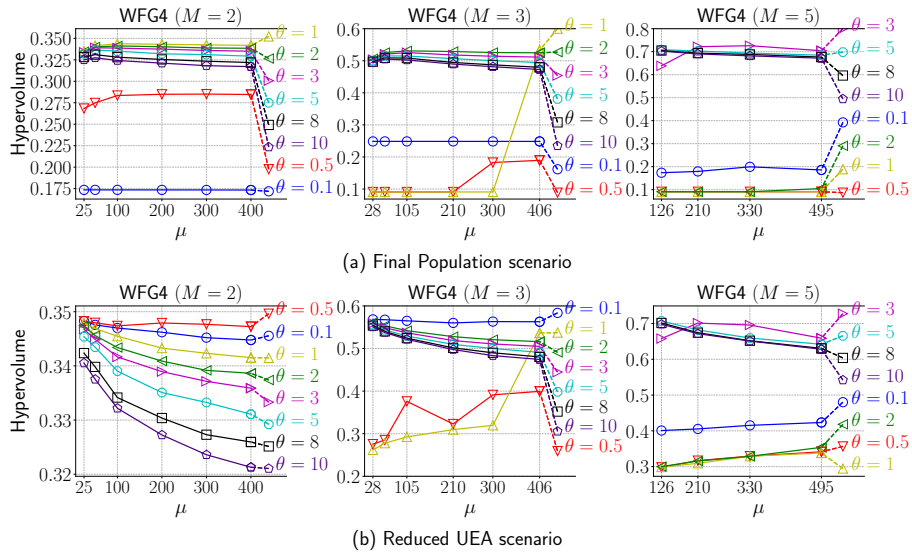


Figure S.73: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

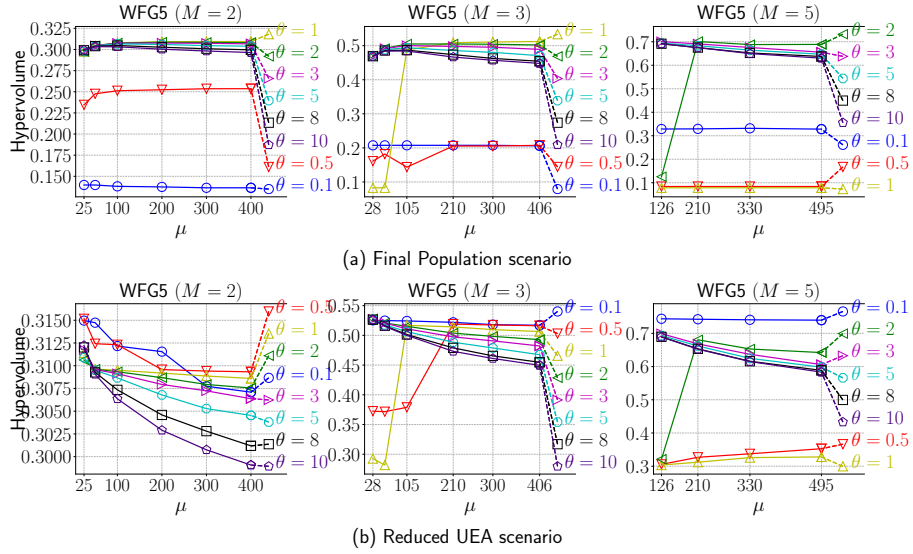


Figure S.74: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG5 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

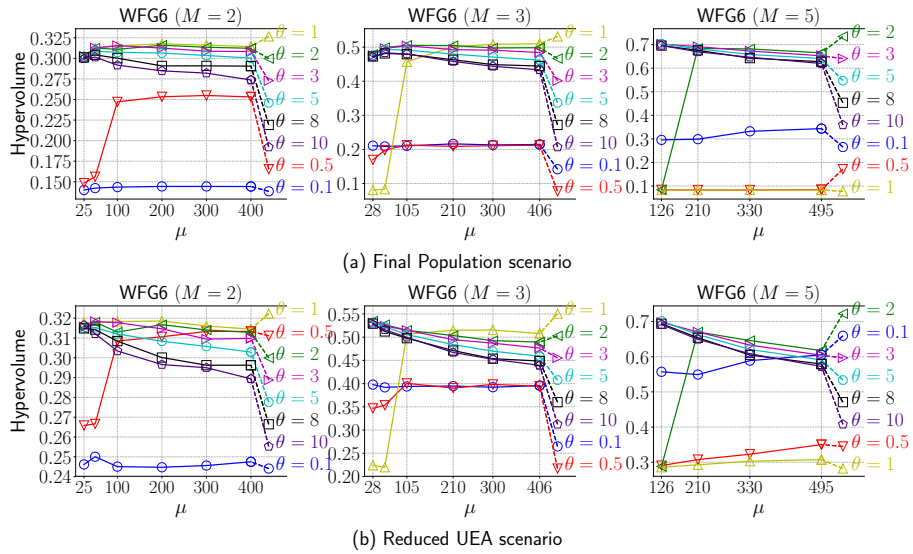


Figure S.75: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG6 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

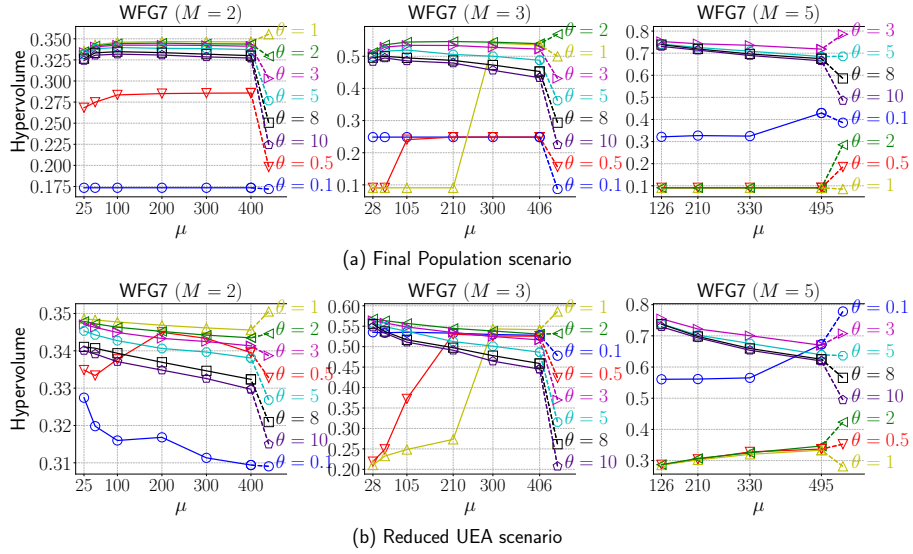


Figure S.76: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG7 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

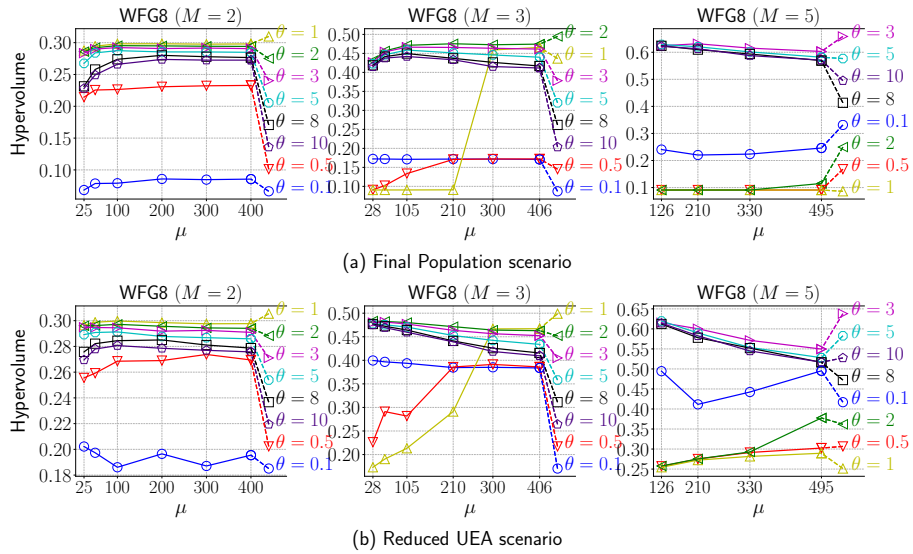


Figure S.77: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG8 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

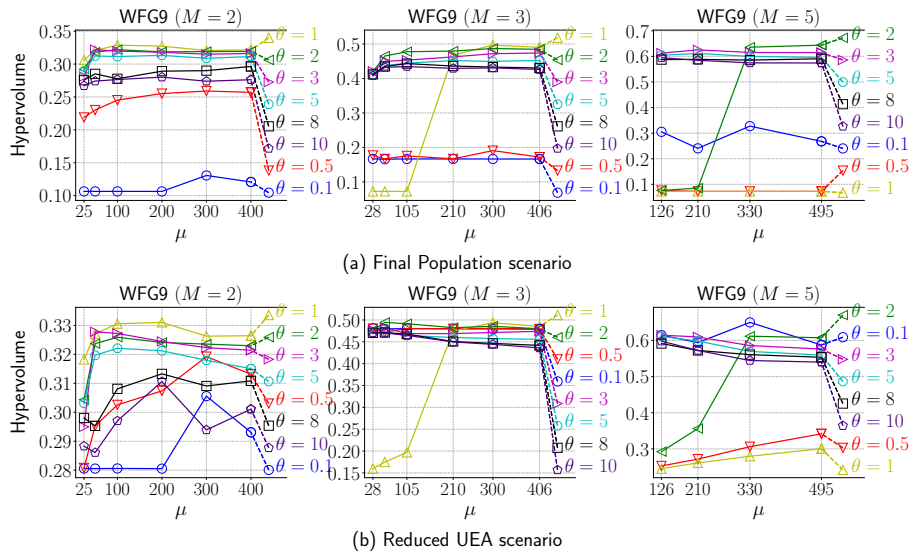


Figure S.78: Influence of μ on the performance of MOEA/D using g^{pbi} with various θ values on the WFG9 problem with $M \in \{2, 3, 5\}$. The median HV value at 50,000 evaluations among 31 runs is shown.

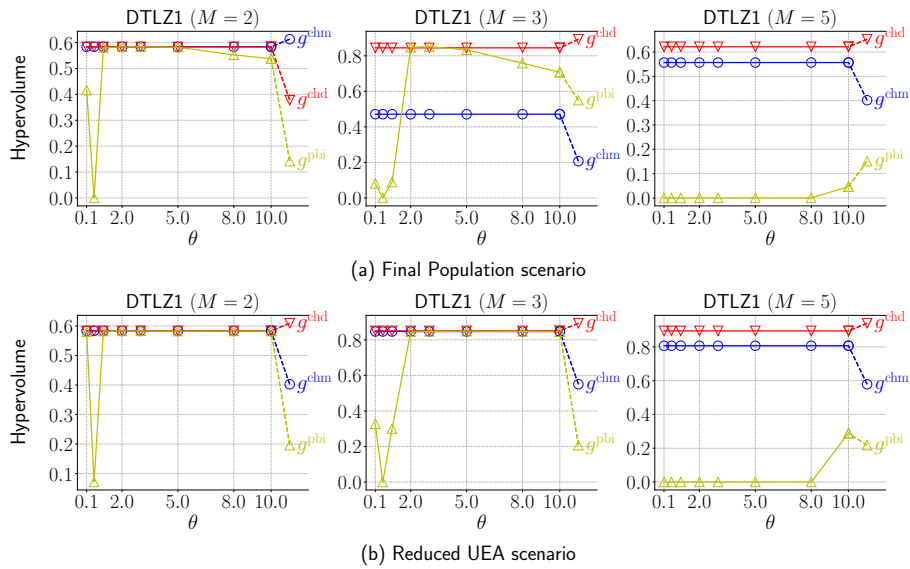


Figure S.79: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the DTLZ1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

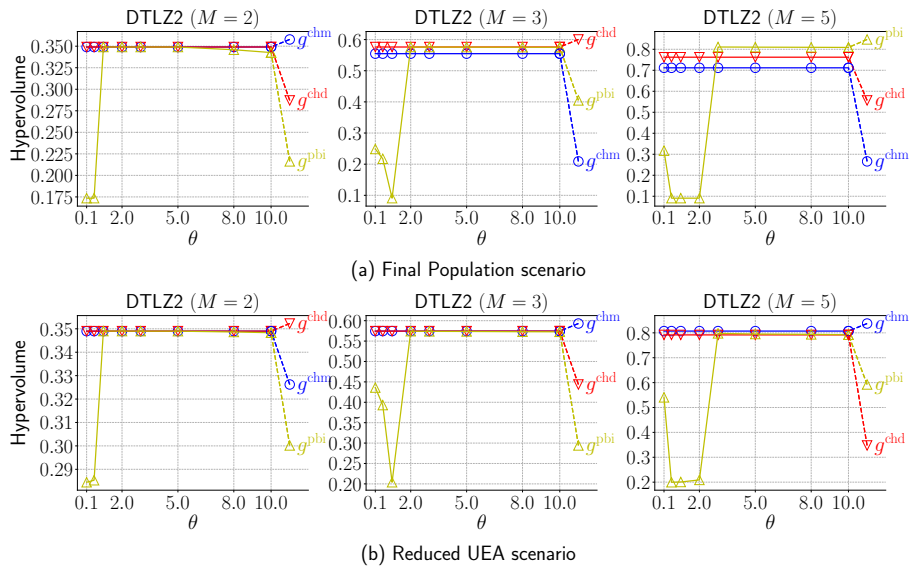


Figure S.80: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the DTLZ2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

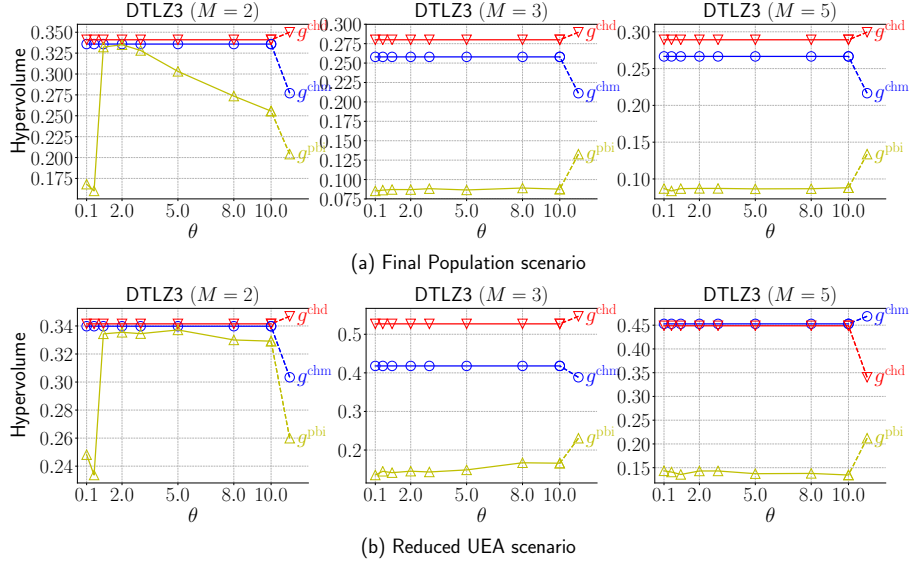


Figure S.81: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the DTLZ3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

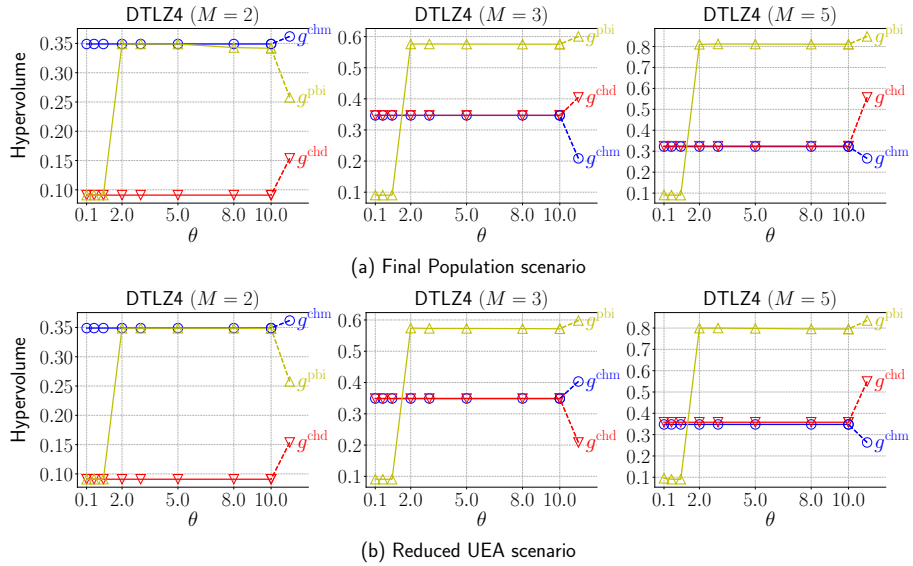


Figure S.82: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the DTLZ4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

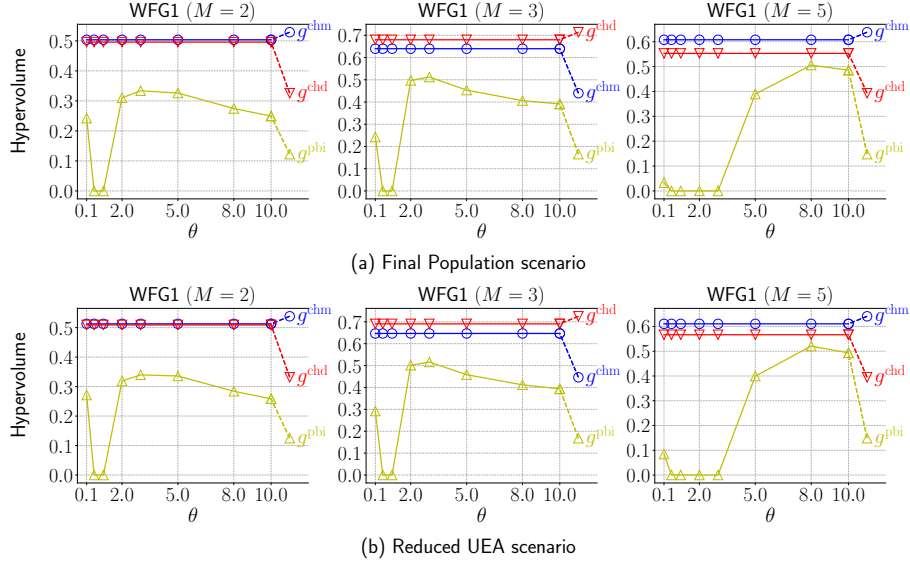


Figure S.83: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG1 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

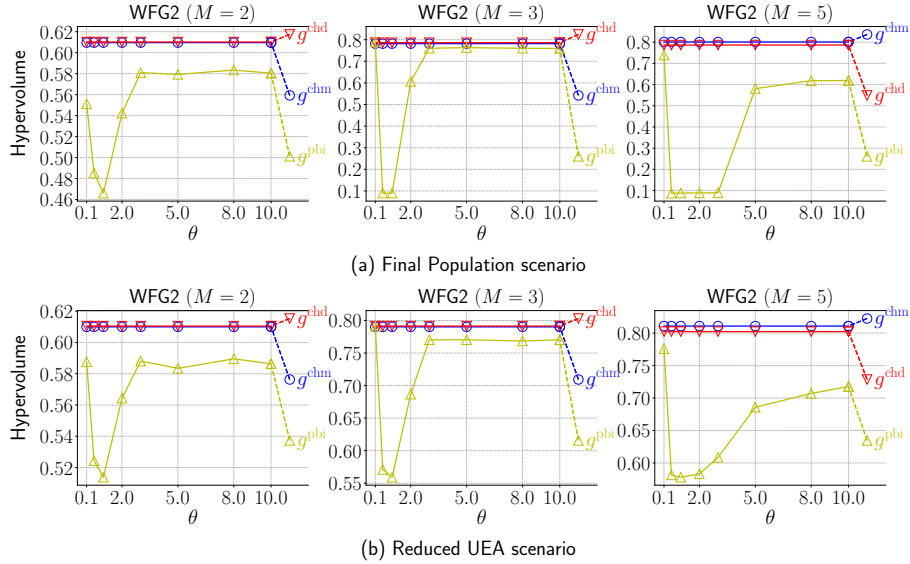


Figure S.84: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG2 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

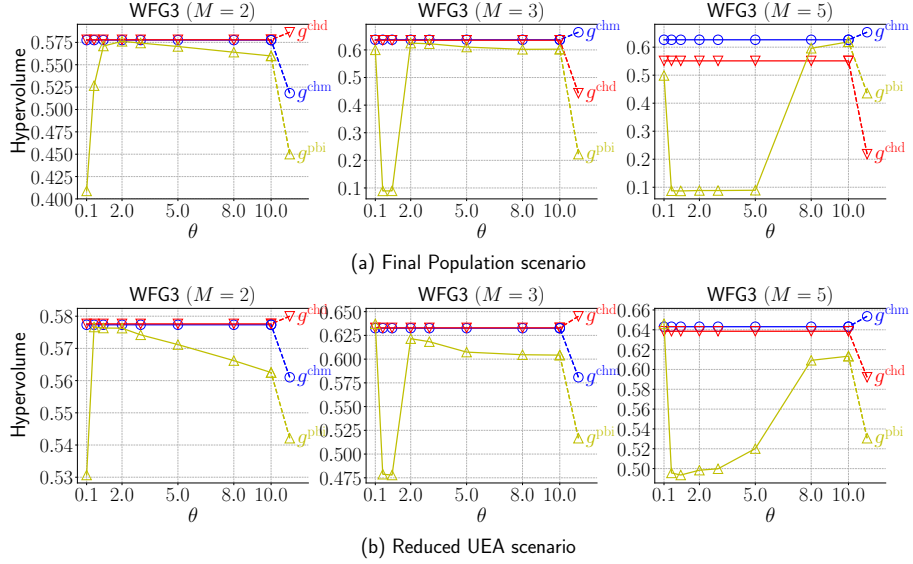


Figure S.85: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG3 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

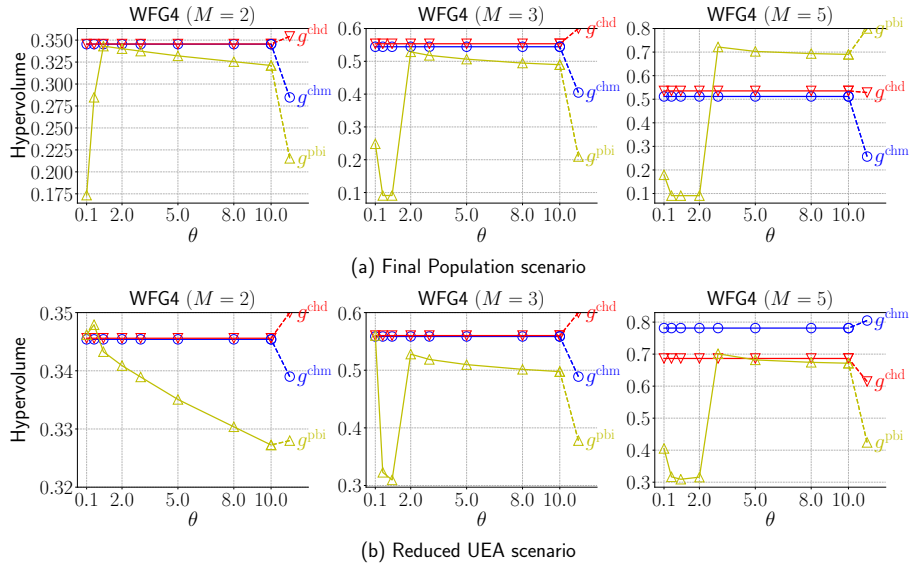


Figure S.86: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG4 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

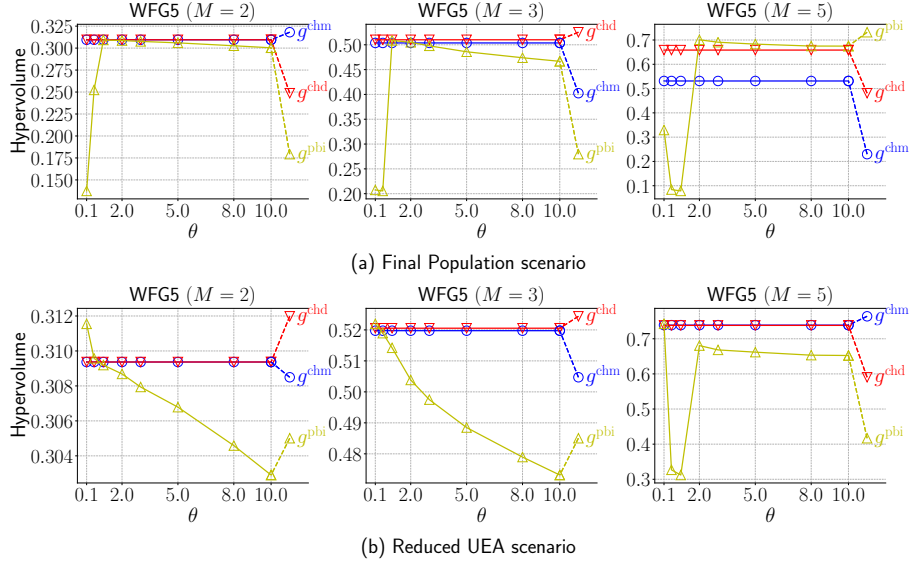


Figure S.87: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG5 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

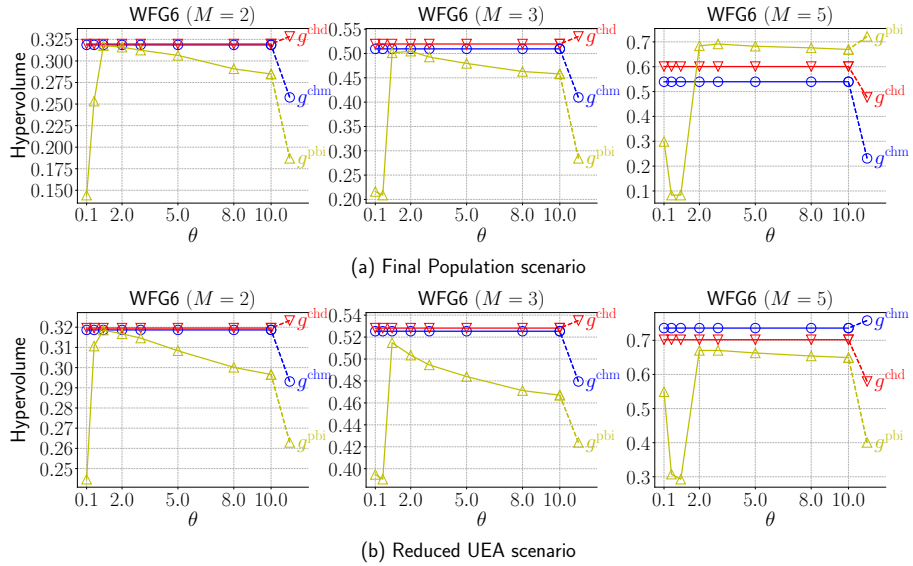


Figure S.88: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG6 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

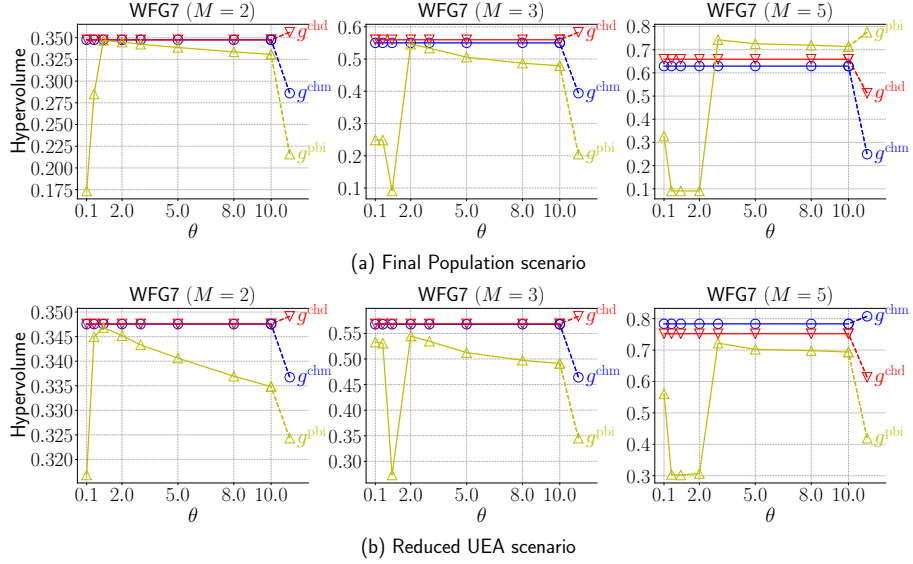


Figure S.89: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG7 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

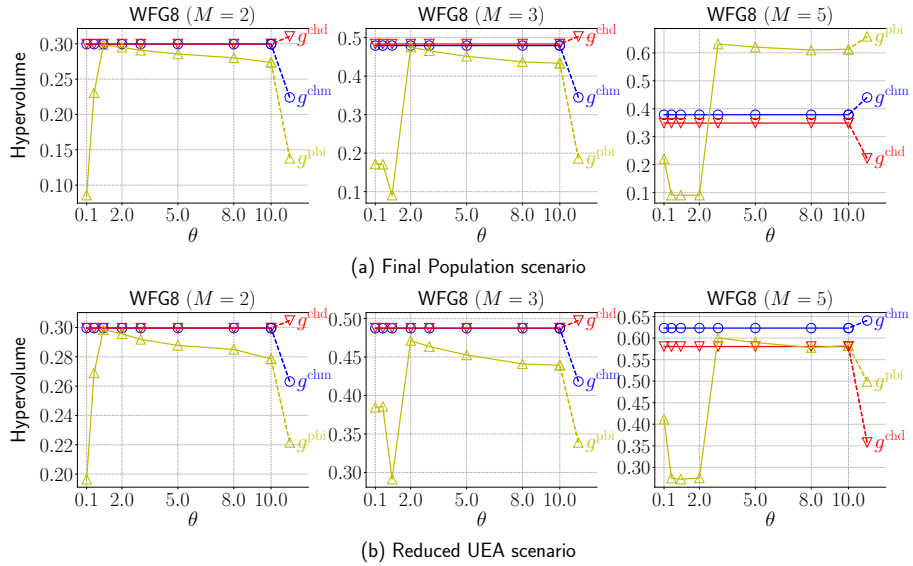


Figure S.90: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG8 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.

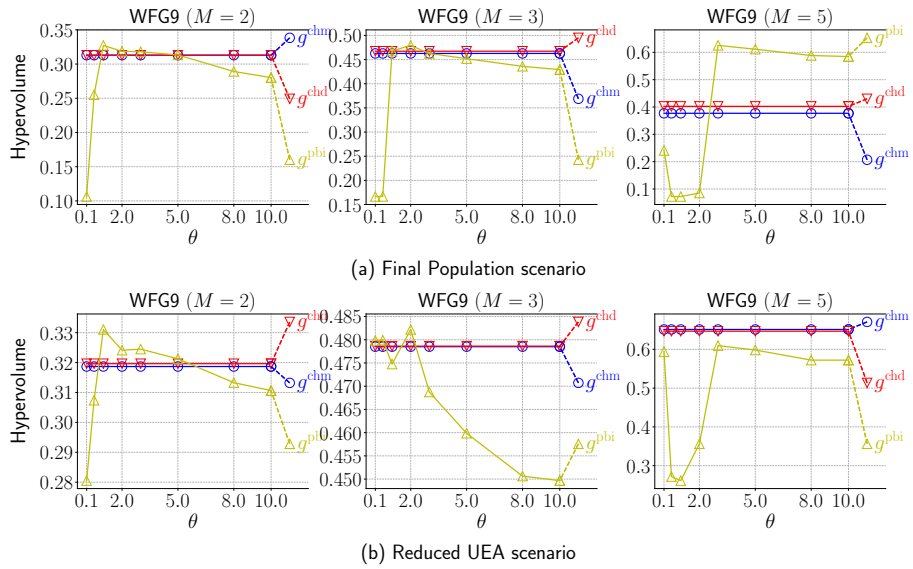


Figure S.91: Comparison of the two Chebyshev functions (g^{chm} and g^{chd}) and g^{pbi} with various θ values on the WFG9 problem with $M \in \{2, 3, 5\}$. The median HV value at 50 000 evaluations among 31 runs is shown.